# Math 1312 Section 3.3 Analyzing Isosceles Triangles

## **Definitions:**

An **isosceles triangle** is a triangle having at least two congruent (of equal length) sides. The two sides are called the **legs** and the third side is called the **base**.

The point at which the legs meet is the **vertex** and the angle there is the **vertex angle**.

The two angles that include the base are called the **base angles**.



**Example:** Name the parts of this isosceles triangle:



### **Other important triangle parts:**

### **Definitions:**

- Median is a segment that starts from an angle and goes to the <u>midpoint</u> of the opposite side.
- > Altitude is a segment that starts from an angle and is perpendicular to the opposite side.
- Angle bisector of a triangle is a segment that <u>bisects an angle and goes to the opposite</u> side.
- Perpendicular bisector is a segment that <u>passes through the midpoint</u> of a side AND is <u>perpendicular to that side</u>.

**Example:** *Fill in the blanks.* 

- a)  $\overline{DF}$  is \_\_\_\_\_ of  $\Delta DEC$ .
- b)  $\overline{EH}$  is \_\_\_\_\_ of  $\Delta DEC$ .





Theorem: Corresponding altitudes of congruent triangles are congruent.

**Theorem:** The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.

**Isosceles Triangle Theorem:** If two sides of a triangle are congruent, then the angles opposite those sides are congruent.



**AND** (converse): If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



#### **Example:**

 $\Delta DEF$  is isosceles.  $\angle D$  is the vertex angle.  $m \angle E = 2x + 40$  and  $m \angle E = 3x + 22$ . Find the measures of each angle.



Note:

- > A triangle is equilateral **if and only if** it is equiangular.
- $\blacktriangleright$  Each angle of an equilateral triangle measures 60°.

**Definition:** The perimeter of a triangle is the sum of the lengths of all of its sides.

**Example:** In the figure below,  $\overline{PQ} \cong \overline{PR}$ , and  $\overline{PS}$  and  $\overline{ST}$  are medians. Find QT and QR.



**Example:**  $\overline{KL}$  is an altitude of  $\Delta HJK$ . Find x.



**Example:**  $\overline{PO}$  is the perpendicular bisector of  $\overline{MN}$ . Find x.



**Example:** In  $\Delta JKL$ ,  $\overline{JK} \cong \overline{JL}$ , and  $\overline{JM}$  is both a median, and altitude, and an angle bisector. Find the following.

- a) *m∠KMJ*
- b) *KL*
- c) *m∠KJM*

d) *m∠KJL* 



e) *m∠K* 

Example:

a)  $x = \_$ \_\_\_\_





c) 
$$x =$$
\_\_\_\_\_



**Example:** Use the figure below to find the angle measures if  $m \angle 1 = 30$ .

