Math 1312  
Section 3.3  
Analyzing Isosceles Triangles

Definitions:
An isosceles triangle is a triangle having at least two congruent (of equal length) sides. The two sides are called the legs and the third side is called the base. The point at which the legs meet is the vertex and the angle there is the vertex angle. The two angles that include the base are called the base angles.

Example: Name the parts of this isosceles triangle:

Other important triangle parts:

Definitions:
- **Median** is a segment that starts from an angle and goes to the midpoint of the opposite side.
- **Altitude** is a segment that starts from an angle and is perpendicular to the opposite side.
- **Angle bisector** of a triangle is a segment that bisects an angle and goes to the opposite side.
- **Perpendicular bisector** is a segment that passes through the midpoint of a side AND is perpendicular to that side.
Example: Fill in the blanks.

a) $\overline{DF}$ is ____________ of $\triangle DEC$.

b) $\overline{EH}$ is ____________ of $\triangle DEC$.

c) $\overline{RV}$ is ____________ of $\triangle RST$.

d) $\overline{WZ}$ is ____________ of $\overline{RS}$.

Theorem: Corresponding altitudes of congruent triangles are congruent.
**Theorem:** The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.

**Isosceles Triangle Theorem:** If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

**AND (converse):** If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

**Example:**
\( \triangle DEF \) is isosceles. \( \angle D \) is the vertex angle. \( m\angle E = 2x + 40 \) and \( m\angle E = 3x + 22 \). Find the measures of each angle.
Note:
- A triangle is equilateral if and only if it is equiangular.
- Each angle of an equilateral triangle measures 60°.

Definition: The perimeter of a triangle is the sum of the lengths of all of its sides.

Example: In the figure below, $\overline{PQ} \cong \overline{PR}$, and $\overline{PS}$ and $\overline{ST}$ are medians. Find QT and QR.

Example: $\overline{KL}$ is an altitude of $\triangle HJK$. Find $x$.
Example: \( PO \) is the perpendicular bisector of \( MN \). Find \( x \).

\[ 2x + 5 \quad 15 - 3x \]

Example: In \( \triangle JKL \), \( JK \equiv JL \), and \( JM \) is both a median, and altitude, and an angle bisector. Find the following.

a) \( m\angle KMJ \)

b) \( KL \)

c) \( m\angle KJM \)

d) \( m\angle KJL \)

e) \( m\angle K \)
Example:
a) \( x = \) __________

b) \( x = \) __________

c) \( x = \) __________
**Example:** Use the figure below to find the angle measures if $m\angle 1 = 30$.

$m\angle 2 = \underline{\hspace{2cm}}$

$m\angle 3 = \underline{\hspace{2cm}}$

$m\angle 4 = \underline{\hspace{2cm}}$

$m\angle 5 = \underline{\hspace{2cm}}$

$m\angle 6 = \underline{\hspace{2cm}}$

$m\angle 7 = \underline{\hspace{2cm}}$

$m\angle 8 = \underline{\hspace{2cm}}$