Section 6.2
Definition of Probability

The ratio $\frac{m}{n}$ is the **relative frequency** of an event E that occurs *m* times after *n* repetitions.

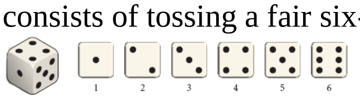
Note: The probability of an event is a number that lies between 0 and 1, inclusive.

Example 1: A survey was taken regarding the number of miles a person drives to work each day. It was found that 534 people drive 15 miles or less to work each day, 308 people drive 16 to 25 miles to work each day, and the rest of the group drive more than 25 miles to work each day. If 3000 people were surveyed, what is the probability that a person surveyed drives more than 25 miles to work each day?

Sample Spaces, Simple Events and Probability Distributions

If $S = \{s_1, s_2, \ldots, s_n\}$ is a finite sample space with *n* outcomes, then the events $\{s_1\}$, $\{s_2\}, \ldots, \{s_n\}$ are the **simple events** of the experiment.

For example, recall the following example from the previous section: An experiment consists of tossing a fair six-sided die, and recording the outcomes of the experiment.



The finite sample space is S = {1, 2, 3, 4, 5, 6}, so {1}, {2}, {3}, {4}, {5} and {6} are the simple events of the experiment.

If each of these simple events is assigned a probability, then we obtain a probability distribution.

A **probability distribution** assigns to each simple event of an experiment a probability.

Example 2: The accompanying data were obtained from a survey of 1,500 people who were asked: "What type of vehicle do you drive?"

Type of Vehicle	Number of Respondents
A – SUV	795
B – Car	480
C – Truck	120
D – Mini Van	105

a. Give an appropriate sample space.
b. Find the probability distribution associated with this experiment.

Given a general finite sample space: S = {s_1, s_2,…, s_n}, the simple probabilities are denoted as: P({s_1}), P({s_2}),…, P({s_n}) and they have the following properties:

I. $0 \le P(s_i) \le 1$, $i \in \{1, 2, \ldots, n\}$
II. $P(s_1) + P(s_2) + \ldots + P(s_n) = 1$

Finding the probability of an Event E
1. Determine the sample space S.
2. Assign probabilities to each of the simple events of S.
3. If E = {s_1, s_2,…, s_k} ($k \neq n$), then $P(E) = P(s_1) + ... + P(s_k)$.

Example 3: Let $S = \{s_1, s_2, s_3, s_4, s_5\}$ be the sample space associated with an experiment having the following probability distribution:

Outcome	s_1	s_2	s_3	s_4	s_5
Probability	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Find the probability of the event:
a. $G = \{s_1, s_2\}$

b. $H = \{s_1, s_2, s_4, s_5\}$
Recall: $P(s_1) + P(s_2) + \ldots + P(s_n) = 1$

c. $F = \emptyset$

Uniform Sample Spaces

A sample space in which the outcomes of an experiment are equally likely to occur is called a **uniform sample space.**

Let S={s_1, s_2,…, s_n} be a uniform sample space. Then $P(s_1) = P(s_2) = ... = P(s_n) = \frac{1}{n}$.

Example 4: An experiment consists of tossing a pair of fair dice and observing the number that falls uppermost. The sample space follows:

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
⚁	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
⚂	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
⚃	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
⚄	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
⚅	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

a. What is the probability of each outcome?

b. What is the probability that the at least one 3 is cast?

c. What is the probability that the sum of the dice is at most 11?

Some examples will involve the use of the 52-card deck. Assume the Jokers are taken out. The 52-card deck contains the following cards:

2 ♡	3 ♡	4 ♡	5 ♡	6 ♡	7 ♡	8 ♡	9 ♡	10 ♡	J ♡	Q ♡	K ♡	A ♡
2 ♢	3 ♢	4 ♢	5 ♢	6 ♢	7 ♢	8 ♢	9 ♢	10 ♢	J ♢	Q ♢	K ♢	A ♢
2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣	8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	A ♣
2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠	8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	A ♠

Example 5: If one card is drawn from a well-shuffled standard 52-card deck, what is the probability that the card drawn is:

a. a nine?

b. an Ace or a King?

c. not red?