Section 7.4
The Binomial Distribution

A binomial experiment has the following properties:

1. The number of trials is fixed.
2. There are two outcomes of the experiment: Success, with probability p and Failure, with probability q. Note: $p + q = 1$.
3. The probability of success in each trial is the same.
4. The trials are independent of each other.

Experiments with two outcomes are called **Bernoulli trials** or **Binomial trials**.

Finding the Probability of an Event of a Binomial Experiment
In a binomial experiment in which the probability of success in any trial is p, the probability of exactly x successes in n independent trials is given by

$$P(X = x) = C(n,x)\,p^x q^{n-x}$$

X is called a **binomial random variable** and its probability distribution is called a **binomial probability distribution**.

Example 1: An experiment consists of 10 independent trials where the probability of success is $\frac{5}{8}$. Find each of the following probabilities.

a. The probability of obtaining exactly 5 successes.

b. The probability of obtaining at least 1 success.

c. $P(X \le 1)$

Mean, Variance and Standard Deviation of a Random Variable

If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p, and probability of failure q, then the mean E(X), variance and standard deviation of X are given by applying the following formulas:

$$\mu = np$$
$$Var(X) = npq$$
$$\sigma = \sqrt{Var(X)}$$

Example 2: Consider the following binomial experiment. If the probability that a marriage will end in divorce within 20 years after its start is 0.84, what is the probability that out of 6 couples just married, in the next 20 years:

a. none will be divorced?

b. all will be divorced?

c. Find the mean and standard deviation of the experiment.

Example 3: Consider the following binomial experiment. It is estimated that 34% of the general population has blood type A^+. If a sample of 9 people is selected at random, what is the probability that at least 8 of them have blood type A^+?

Example 4: The probability of a person contracting influenza on exposure is 62%. In the binomial experiment for a group of 12 people that has been exposed, what is the probability that at most 10 contract influenza?