Around 30 Questions

110 mins

Math 1313 Final Exam Review

(x_1, y_1) (x_2, y_2)

Example 1: Find the equation of the line containing points (1,2) and (2,3).

$y - y_1 = m(x - x_1)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-2}{2-1} = 1$

$y - 2 = 1(x - 1)$

$y - 2 = x - 1$

$y = x + 1$

$\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

Example 2: The Ace Company installed a new machine in one of its factories at a cost of \$20,000. The machine is depreciated linearly over 10 years with a scrap value of \$2,000. Find the value of the machine after 5 years.

$m = \dfrac{S.V - I.V}{t} = \dfrac{2000 - 20000}{10} = -1800$

$V(t) = mt + I.V = -1800t + 20000$

$V(5) = -1800(5) + 20000 = \$11,000$

Example 3: The AC Florist Company got a new delivery van at a cost of \$28,000. The van is depreciated linearly over 5 years and has no scrap value. Find the value of the machine after 2 years.

$m = \dfrac{0 - 28000}{5} = -5,600$

$V(t) = -5600t + 28000$

$V(2) = -5600(2) + 28000 = \$16,800$

Example 4: 4. A manufacturer has a monthly fixed cost of \$1200 and a production cost of \$2.50 for each unit produced. The product sells for \$10 per unit.

a. What is the cost function?

$C(x) = Cx + F.C = 2.5x + 1200$

b. What is the revenue function?

$R(x) = Sx = 10x$

c. What is the profit function?

$P(x) = R(x) - C(x) = 10x - 2.5x - 1200 = 7.5x - 1200$

d. What is the break- even point?

$P(x) = 0$ (OR $R(x) = C(x)$)

$7.5x - 1200 = 0$

$x = 160$

B.E quantity

$(160, 1600)$ ← B.E point

(*) Find profit/Loss if 150 items are sold

$7.5(150) - 1200 = -75$ Loss of \$75

Example 5: Solve using Gauss-Jordon.

$$\begin{aligned} x - 5y + z &= 5 \\ -y + z &= 2 \\ 3x + 2y + z &= 11 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & -1 & 1 & 2 \\ 3 & 2 & 1 & 11 \end{array}\right) \xrightarrow{R_3 = -3R_1 + R_3} \left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & -1 & 1 & 2 \\ 0 & 17 & -2 & -4 \end{array}\right)$$

$$\xrightarrow{R_2 = -1R_2} \left(\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 17 & -2 & -4 \end{array}\right)$$

$$\xrightarrow[R_3 = -17R_2 + R_3]{R_1 = 5R_2 + R_1} \left(\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 15 & 30 \end{array}\right) \xrightarrow{R_3 = \frac{1}{15}R_3} \left(\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & \boxed{1} & 2 \end{array}\right)$$

$$\xrightarrow[R_2 = R_3 + R_1]{R_1 = 4R_3 + R_1} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right) \end{array}$$

$x = 3$
$y = 0$
$z = 2$

Example 6: Given the following matrices are in row reduced form. State the solution, if it exists to the system of equations.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 6 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

$x = -2$, $y = 1$, $z = 6$ — unique sol^n

Inf sol^n: $x = 2$, $y + 2z = 0$

$z = -1$, $y = 3$, $\underline{0 = 4}$ X — No sol^n

Example 7: Solve for a, b, c and d.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} a-2 & b+1 \\ c+5 & d-6 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -2 & 4 \end{bmatrix} \Rightarrow \begin{aligned} a - 2 &= 4 \\ b + 1 &= 3 \\ c + 5 &= -2 \\ d - 6 &= 4 \end{aligned} \qquad \begin{aligned} a &= 6 \\ b &= 2 \\ c &= -7 \\ d &= 10 \end{aligned}$$

Example 8: Find the transpose of the following matrices.

$$A = \begin{bmatrix} -2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 2 & 1 \\ 0 & -1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -2 & 1 \\ 3 & 0 \\ 2 & 4 \end{bmatrix} \qquad B^T = \begin{bmatrix} 2 & 0 & 1 \\ 2 & -1 & 2 \\ 1 & 3 & 4 \end{bmatrix}$$

Popper #31 → 1-5: B

Example 9: Find the product, if possible.

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \\ -1 & 2 \end{bmatrix}$$

(2×4) (3×2) Not possible

Example 10: Find the product, if possible

$$\begin{bmatrix} 2 & 1 \\ 1 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -1 \end{bmatrix}$$

$(3\times 2)(2\times 3)$ 3×3

$$\begin{pmatrix} 2\cdot 1+1\cdot 3 & 2\cdot -1+1\cdot 0 & 2\cdot 2+1\cdot -1 \\ 1\cdot 1+0\cdot 3 & 1\cdot -1+0\cdot 0 & 1\cdot 2+0\cdot -1 \\ -1\cdot 1+2\cdot 3 & -1\cdot -1+2\cdot 0 & -1\cdot 2+2\cdot -1 \end{pmatrix} = \begin{pmatrix} 5 & -2 & 3 \\ 1 & -1 & 2 \\ 5 & 1 & -4 \end{pmatrix}$$

Example 11: Find the inverse of the following matrix

$$\begin{bmatrix} 5 & 3 \\ -4 & 6 \end{bmatrix}$$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$D = 5\cdot 6 - 3\cdot(-4) = 42 \neq 0$

$A^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $D = ad - bc \neq 0$

$\frac{1}{42}\begin{bmatrix} 6 & -3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 1/7 & -1/14 \\ 2/21 & 5/42 \end{bmatrix}$

Example 12: A manufacturer of stereo speakers, makes two kinds of speakers, an economy model which sells for \$50 and a deluxe model which sells for \$200. The deluxe model uses 1 woofer and 2 tweeters. The economy uses 1 woofer and 1 tweeter. The manufacturer currently has 20 woofers and 45 tweeters in inventory. Set-up the problem to maximize income from the sale, use x for economy and y for deluxe.

	E (x)	D (y)	
W	1	1	≤ 20
T	1	2	≤ 45
I	50	200	

Max: $50x + 200y$

subject to: $x + y \leq 20$

$x + 2y \leq 45$

$x \geq 0,\ y \geq 0$

$2x + 3y = 12$

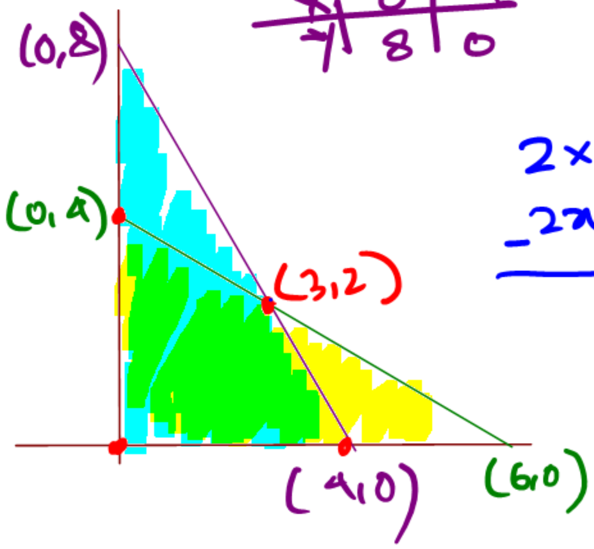
x	0	6
y	4	0

Example 13: Maximize the following Linear Programming Problem.

$2x + y = 8$

x	0	4
y	8	0

$$\begin{aligned} Max\ P &= 3x + 2y \\ st:\ 2x + 3y &\le 12 \\ 2x + y &\le 8 \\ x, y &\ge 0 \end{aligned}$$



$$\begin{aligned} 2x + 3y &= 12 \\ -\ 2x \pm y &= 8 \\ \hline 2y &= 4 \\ y &= 2 \\ x &= 3 \end{aligned}$$

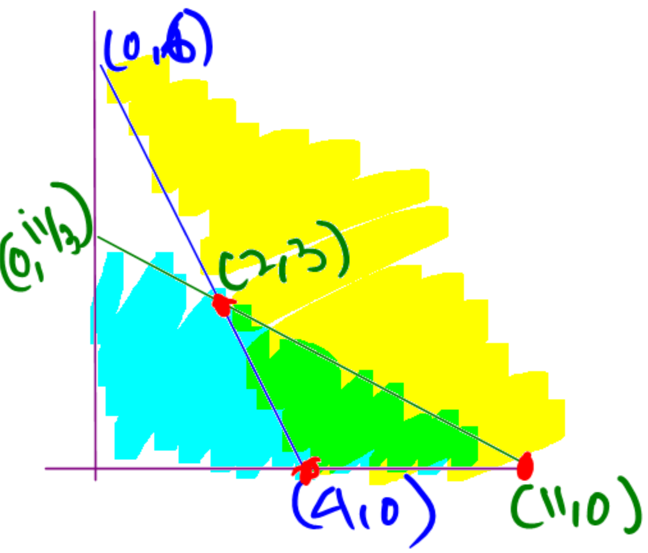
pts	$3x+2y$
(0,4)	8
(4,0)	12
(0,0)	0
$\boxed{(3,2)}$	$9 + 4 = \boxed{13}$

Pt

Optimal Value

Example 14: Minimize the following Linear Programming Problem.

$$\begin{aligned} Min\ C &= x + y \\ st:\ 3x + 2y &\ge 12 \\ x + 3y &\le 11 \\ x, y &\ge 0 \end{aligned}$$



$3x + 2y = 12$

x	0	4
y	6	0

	$x+y$
(2,3)	5
(4,0)	4 ⇐
(11,0)	11

$x + 3y = 11$

x	0	0
y	11/3 ~4	11

$$\begin{aligned} 3x + 2y &= 12 \\ -\ 3x + 9y &= 33 \\ \hline -7y &= -21 \quad y = 3 \end{aligned}$$

Example 15: Find the accumulated amount at the end of 6 months on a \$2000 bank deposit paying simple interest at a rate of 3% per year.

F. SI

$F = P(1 + rt)$

$t = \frac{6}{12} = 0.5$

$$\begin{aligned} F &= 2000(1 + (.03)(0.5)) \\ &= 2000(1 + 0.15) \\ &= 2300 \end{aligned}$$

$i = \frac{r}{m}$

$n = mt$

Example 16: Dave invested a sum of money 3 years ago in a savings account that has since paid interest at the rate of 4.5% per year compounded monthly. His investment is now worth \$5,721.24. How much did he originally invest?

$m=12$ PV CI

$n = 3 \cdot 12 = 36$ $i = \frac{0.045}{12}$

$P = F(1+i)^{-n}$

$= 5721.24(1+\frac{0.045}{12})^{-36}$

$= \$5000$

Example 17: Mike pays \$300 per month for 4 years for a car, making no down payment. If the loan borrowed costs 7% per year compounded monthly, what was the original cost of the car? How much interest will be paid?

PV A

$i = \frac{0.07}{12}$

$n = 48$

$P = E\left[\frac{1-(1+i)^{-n}}{i}\right]$

$= 300\left[\frac{1-(1+0.07/12)^{-48}}{0.07/12}\right] = \$12528.06$

(300×48)

-12528.06

$= \$1871.94$

Example 18: Steve bought a car for \$30,000 . He put down 10% and financed the balance. His bank charged him 5% compounded monthly for 5 years. What is the monthly payment?

E ?

Amort. $i = \frac{0.05}{12}$

$E = \frac{Pi}{(1-(1+i)^{-n})} = \$509.52$

$n = 60$ $P = (0.9)30000 = 27000$

Example 19: George decided to deposit \$4,000 to pay for a cruise he plans to take in 2 years. His bank pays 3.5% annual interest compounded semiannually. How much will he have in his account at the end of two years?

FV CI

$i = \frac{0.035}{2}$ $n = 4$

$F = P(1+i)^n$

$= 4000(1+\frac{0.035}{2})^4 = \$4287.44$

Example 20: Sandy decided to save some money for her daughter's college education. She decided to save \$500 per quarter. Her credit union pays 4.5% annual interest compounded quarterly. How much money will she have available when her daughter starts college in 10 years?

E is known ⇒ Annuity

$i = \frac{0.045}{4}$

F.V A

$n = 40$

$F = E\left[\frac{(1+i)^n - 1}{i}\right] = \$25,083.42$

Example 21: Let U={1,2,3,4,5,6,7,8,9,10},A = {1,3,5,7,9}, B={2,4,6,8,10}, C={1,2,4}

a. $B \cap C^c$ $= \{2,4,6,8,10\} \cap \{3,5,6,7,8,9,10\}$

$= \{6,8,10\}$

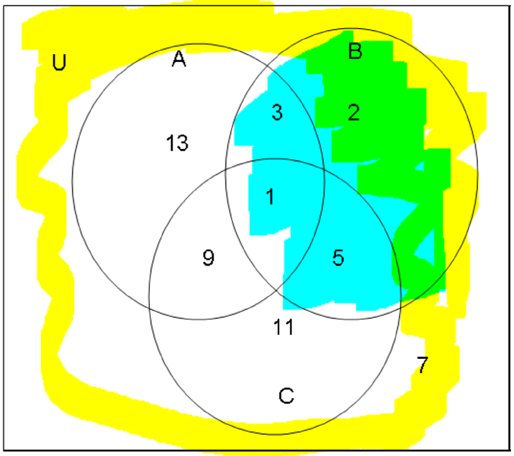
b. $A \cup B^c$

$= \{1,3,5,7,9\} \cup \{1,3,5,7,9\}$

$= \{1,3,5,7,9\}$

$A^c \cap C^c = (A \cup C)^c$

Example 22: Given the Venn Diagram.



a. $n[B \cup (A^C \cap C^c)]$

$= n[B \cup (A \cup C)^c]$

$= 1 + 3 + 5 + 2 + 7$

$= 18$

b. $n[B \cap (A^C \cap C^c)]$

$= 2$

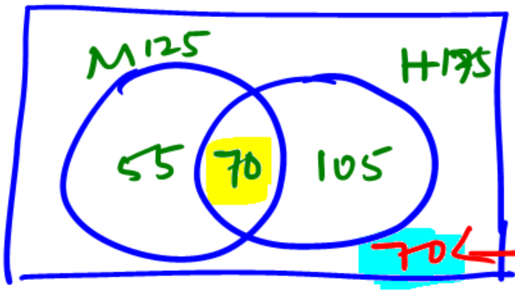
(c). $n[B^c \cup (A \cup C)]$

$= 13 + 3 + 9 + 5 + 11 + 1 + 7$

d) $n[B^c \cap (A \cup C)]$

$= 13 + 9 + 11$

Example 23: In a group of 300 hundred students, 125 are currently taking a math class and 175 are taking a history class and 70 are taking both classes. How many students in this group are taking a math class or a history class but not both?.



Only Math or Hist $= 55 + 105$
$= 160$

At most one of them $= 55 + 105 + 70$
$= 230$

$300 - (55 + 70 + 105)$

At least one $= 55 + 105 + 70$
$= 230$

Example 24: Suppose a person planning a banquet cannot decide how to seat 6 honored guests at the head table. In how many arrangements can they seated in the 6 chairs on one side of the table?

$\underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 6!$ $P(6,6)$ 720

Example 25: In how many ways can a president, vice president, secretary, and treasurer be selected from an organization of 20 members?

$P(20,4)$ 116280

Example 26: You are going to make a serial number which can have no repeats and contains 3 digits and two letters. A zero cannot be the first digit. How many serials numbers are possible?

0 – 9

$\underline{9} \cdot \underline{9} \cdot \underline{8} \cdot \underline{26}\ \underline{25}$ 421200

No 0

Example 27: A car dealer is offering special pricing on a truck. It has four models, six exterior colors, 3 interior colors, four choices of seat coverings and 3 stereo systems. If you can only choose one in each category, how many different trucks could be constructed?

$\underset{\text{Model}}{\underline{4}} \quad \underset{\text{Ext}}{\underline{6}} \quad \underset{\text{Int}}{\underline{3}} \quad \underset{\text{Seat}}{\underline{4}} \quad \underset{\text{St.}}{\underline{3}} = 864$

Example 28: Find the number of ways in which 8 members of the space shuttle crew can be selected from 20 available astronauts.

(a) $C(20, 8) = 125{,}970$ No ordering

b. The command structure on a space flight is determined by the order in which astronauts are selected for the flight. How many different command structures are possible if 8 astronauts are selected from 20 that are available?

ordering Matters

$P(20, 8) = 5079110400$

c. If 14 men and 6 women are available for a space shuttle flight, in how many crews are possible that have 5 men and 3 women?

$C(14,5) \cdot C(6,3) = 40{,}040$

Example 29: A box contains 2 red marbles and 3 black marbles. Two marbles are drawn in succession without replacement. Find the following:

1st Draw — 2nd Draw

$2/5 - R_1$: R_2 $1/4$; B_2 $3/4$

$3/5 - B_1$: R_2 $2/4$; B_2 $2/4$

a. Find the probability the second marble is red?

$P(2^{nd}\text{Red}) = \frac{2}{5} \cdot \frac{1}{4} + \frac{3}{5} \cdot \frac{2}{4} = \frac{8}{20} = 0.4$

b. Find the probability that both marbles are the same color?

$P(\text{both same color}) = (R_1 R_2) + (B_1 B_2) = \frac{2}{5} \cdot \frac{1}{4} + \frac{3}{5} \cdot \frac{2}{4} = 0.4$

c. Find the probability that the second marble is black given the first marble is red?

$P(B_2 | R_1) = \frac{P(B_2 \cap R_1)}{P(R_1)} = \frac{\frac{2}{5} \cdot \frac{3}{4}}{2/5} = \frac{3}{4} = 0.75$

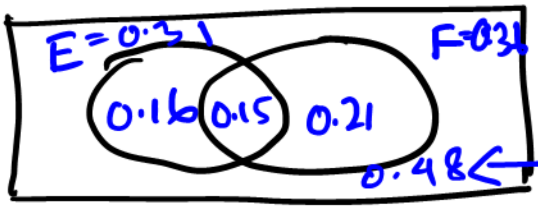
d. Find the probability that first marble red given that the second marble is red?

$P(R_1 | R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)} = \frac{\frac{2}{5} \cdot \frac{1}{4}}{\frac{2}{5} \cdot \frac{1}{4} + \frac{3}{5} \cdot \frac{2}{4}} = \frac{\frac{2}{20}}{\frac{8}{20}} = \frac{2}{8} = 0.25$

Example 30: Let E and F be events of a sample space S. Let $P(E^C) = 0.69$, $P(F) = 0.36$ and $P(E \cap F) = 0.15$. Find $P(E \cup F)$.

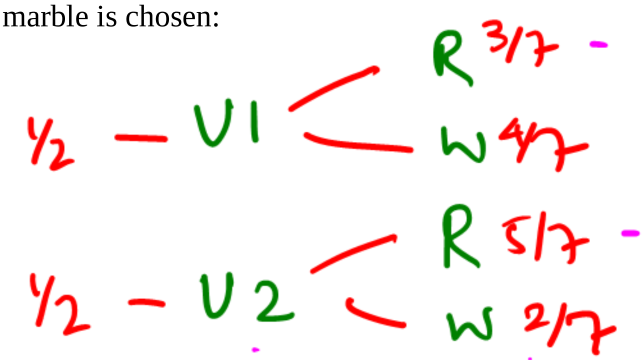
$P(E) = 1 - P(E^c) = 1 - 0.69 = 0.31$

$P(E \cap F) = 0.15$



$1 - (0.16 + 0.15 + 0.21)$ $P(E \cap F)^c = 0.85$

Example 31: Urn I contains 3 red and 4 white marbles and Urn II contains 5 red and 2 white marbles. Each Urn has an equally likely probability of being chosen. Find the following probabilities if a marble is chosen:



a. What is the probability that Urn I is selected and a red marble?

$P(U1 \cap R) = \frac{1}{2} \cdot \frac{3}{7} = \frac{3}{14} = 0.214$

b. What is the probability that a red marble is chosen?

$P(R) = P(U1 \cap R) + P(U2 \cap R) = \frac{1}{2}\frac{3}{7} + \frac{1}{2} \cdot \frac{5}{7} = \frac{8}{14} = 0.5714$

c. What is the probability that Urn I is selected given that a red marble has been selected?

$P(U1 | R) = \frac{P(U1 \cap R)}{P(R)} = \frac{3/14}{8/14} = \frac{3}{8} = 0.375$

d. What is the probability that a white marble is chosen given that Urn II was selected?

$P(W | U2) = \frac{2}{7}$ OR $P(W|U_2) = \frac{P(W \cap U_2)}{P(U_2)} = \frac{\frac{1}{2} \cdot \frac{2}{7}}{\frac{1}{2}} = \frac{2}{7}$

Example 32: 30. A sample of 6 fuses is drawn from a lot containing 10 fuses and 2 defective fuses. Find the probability that the number of defective fuses is:

12: 10G, 2D

a. Exactly 1? $\frac{C(2,1)\,C(10,5)}{C(12,6)} = 0.5455$

b. No defective fuses?

$\frac{C(2,0) \cdot C(10,6)}{C(12,6)} = 0.2273$

c. At least 1 defective fuses?

1 or 2

$\frac{C(2,1) \cdot C(10,5)}{C(12,6)} + \frac{C(2,2)\,C(10,4)}{C(12,6)}$

OR
1 − 0 Def
1 − 0.2273
= 0.7727

Example 33: The probability distribution for a random variable X is given below. Calculate the expected value.

X	14	16	18	20
P(X=x)	0.34	0.31	0.26	0.09

$E(X) = 14(0.34) + 16(0.31) + 18(0.26) + 20(0.09) = 16.2$

Example 34: Consider the following Binomial experiment. The probability that a new employee at a manufacturing plant is still employed after one year is 0.9. Seven people have recently been hired by the company. $p = 0.9 \quad q = 0.1 \quad n = 7$

$P(X=x) = C(n,x)\, p^x q^{n-x}$

a. What is the probability that exactly 4 of these new employees will still be employed after one year?

$x = 4 \quad P(X=4) = C(7,4)(0.9)^4 (0.1)^3 = 0.02296$

b. What is the probability that at least 6 of the new employee's will still be employed after one year? $P(X \geq 6) = P(X=6) + P(X=7)$

$= C(7,6)(0.9)^6(0.1)^1 + C(7,7)(0.9)^7(0.1)^0 = 0.85031$

c. Calculate the mean of new employees that will still be employed after one year?

$\mu = E(X) = np = 7(0.9) = 6.3$

d. Calculate the standard deviation.

$\sigma = \sqrt{Var(X)} = \sqrt{n \cdot p \cdot q} = \sqrt{7(0.9)(0.1)} = 0.7937$

Example 35: Z is a standard normal random variable.

a. Calculate $P(Z > 0.19)$. $= P(Z < -0.19) = 0.4247$

b. Calculate $P(-2.07 < Z < -1.63)$.

$= P(Z < -1.63) - P(Z < -2.07) = 0.0516 - 0.0192$
$= 0.0324$

c. Find the z value, $P(Z > z) = .9115$

$\Rightarrow P(Z < -z) = 0.9115 \Rightarrow -z = 1.35$
$\Rightarrow z = -1.35$

d. Find the value of z, $P(-z < Z < z) = .8444$

$P(Z < z) = \frac{1}{2}(1 + P(-z < Z < z))$

$= \frac{1}{2}(1 + 0.8444) = 0.9222$

$z = 1.42$

Example 36: Suppose X is a normal random variable with μ = 380 and σ = 20. Find the value of:

a. P(X < 405) $= P\left(Z < \frac{405-380}{20}\right)$

$= P(Z < 1.25) = 0.8944$

$P(330 < X < 405)$

$= P\left(\frac{330-380}{20} < Z < \frac{405-380}{20}\right)$

$= P(-2.5 < Z < 1.25)$

$= P(Z < 1.25) - P(Z < -2.5)$

b. *P*(*X* > 330)

$= P\left(Z > \frac{330-380}{20}\right)$

$= P(Z > -2.5) = P(Z < 2.5) = 0.9938$

Example 37: Use the normal distribution to approximate the following binomial distribution. Consider the random sample of 100 drivers on interstate 10 in Texas, where 29% of the drivers exceed the 70 mph speed limit. Find the probability that fewer than 40 drivers exceed the speed limit.

$n = 100 \quad p = 0.29 \quad q = 1 - p = 0.71$

$\mu = np = 100(0.29) = 29 \quad \sigma = \sqrt{npq} = 4.5376$

$P(X < 40)$

$= P(Y \le 39.5)$

$= P(Y < 39.5)$

$= P\left(Z < \frac{39.5 - \mu}{\sigma}\right)$

$= P\left(Z < \frac{39.5 - 29}{4.5376}\right) = P(Z < 2.3139)$

$= P(Z < 2.31)$

$= 0.9896$