Math 1313
Prerequisites/Test 1 Review

Test 1 (Prerequisite Test) is the only exam that can be done from ANYWHERE online. Two attempts. See "Online Assignments" in your CASA account. Note the deadline too.

On CASA, also under "Online Assignments" you can also find Practice Test 1. Do this for extra credit!

Example 1: Simplify.

$\sqrt{32}$
$= \sqrt{16 \cdot 2}$
$= \sqrt{16} \cdot \sqrt{2}$
$= 4\sqrt{2}$

a. $3 + 12 \div 2 - 5 \cdot \sqrt{32}$ ③ ① ② ④

$= 3 + 6 - 5 \cdot 4\sqrt{2}$
$= 9 - 20\sqrt{2}$

b. $\left(\frac{-2\cdot 10 \div (-1)^3}{(-2)^4 - 4}\right)$

$(-2)^4 = 16$
$-2^4 = -16$

$= \left(\frac{-20 \div (-1)}{16-4}\right)$
$= \frac{20}{12} = \frac{5}{3}$

Equations of Lines

Slope of a line: $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$ The **slope** of a line measures the steepness of a line.

$P_1(x_1, y_2)$
$P_2(x_2, y_2)$

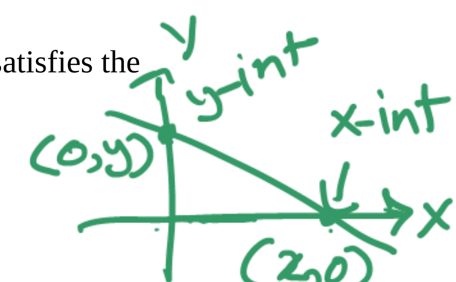
→ Slope-Intercept Form: $y = mx + b$ ← y-int
Point-Slope Form: $y - y_1 = m(x - x_1)$
Standard Form: $ax + by = c$

$2x + 3y = 6$

Example 2: Write an equation of a line, in slope-intercept form, that satisfies the following conditions:
a. has slope 2 and x-intercept 6.

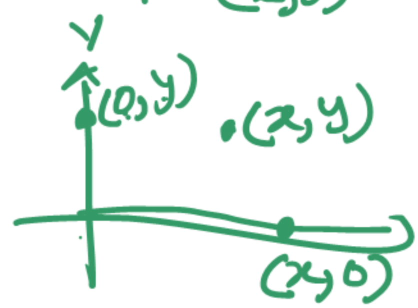
$(6, 0)$
(x_1, y_1)

$y - 0 = 2(x - 6)$
$y = 2x - 12$



b. has slope $-\frac{4}{3}$ and has y-intercept $\frac{9}{2}$.

$y = -\frac{4}{3}x + \frac{9}{2}$



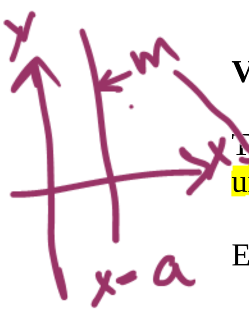
(x_1, y_1) (x_2, y_2)

Example 3: A line passes through (3, -4) and (9, -6). Find:
a. its slope.

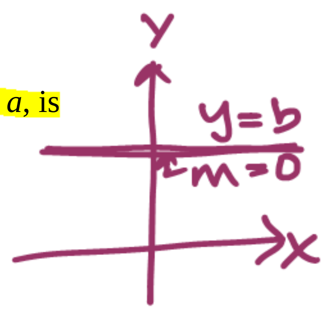
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - (-4)}{9 - 3} = \frac{-6+4}{6} = -\frac{2}{6} = -\frac{1}{3}$$

b. an equation of the line, in slope-intercept form.

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = -\frac{1}{3}(x - 3)$$
$$y + 4 = -\frac{x}{3} + 1$$
$$y = -\frac{x}{3} + 1 - 4 \Rightarrow y = -\frac{x}{3} - 3$$



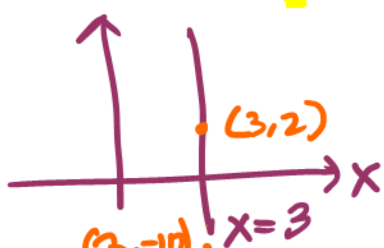
Vertical and Horizontal Lines



The slope of any horizontal line, $y = b$, is 0 and the slope of any vertical line, $x = a$, is undefined.

Example 4: Write an equation of a line that passes through (3, 2) and (3, -10).

$X = 3$

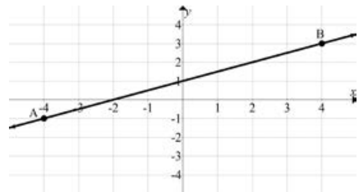


Example 5: Write an equation of a line that has a slope of 0 and passes through (-2, -5).

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 0.(x - (-2))$$
$$y + 5 = 0$$
$$y = -5$$

$y = -5$

Graphs of Lines
The **graph** of a linear equation is a straight line like:



Point of Intersection of Lines
You can use either the substitution method or the elimination method to find any point(s) of intersection of two lines. A system of linear equations will have no solution, one solution or infinitely many solutions.

Example 6: Find any point(s) of intersection.

Sub

a. $2x - 5y = 2$
$x - 4y = -2$

$x = -2 + 4y$

$2(-2 + 4y) - 5y = 2$

$-4 + 8y - 5y = 2$

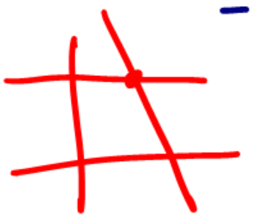
$-4 + 3y = 2$

$\frac{3y}{3} = \frac{6}{3}$

$y = 2$

$x = -2 + 4(2)$
$= -2 + 8 = 6$

(6, 2)



b. $(3x - 4y = 11) \cdot 3$
$(2x + 3y = -4) \cdot 4$

$9x - 12y = 33$
$+ \; 8x + 12y = -16$

$17x = 17$
$x = 1$

$3(1) - 4y = 11$
$3 - 4y = 11$
$-4y = 8$
$y = -2$

(1, -2)

c. $y = 2 - \frac{1}{2}x$
$3x + 6y = 13$

$3x + 6\left(2 - \frac{1}{2}x\right) = 13$

$\cancel{3x} + 12 - \cancel{3x} = 13$

$12 = 13$ X

No sol^n

d. $2x - y = 3$
$\frac{2y - 4x = -6}{2 \quad\; 2 \quad\; 2}$

$(y - 2x = -3) \cdot (-1)$

$2x - y = 3$

Inf sol^n

(0, -3)
(1, -2)

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

Unique (pt of intersection)

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

No' sol^n

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Inft sol^n

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$2x - 5y = 2$

$x - 4y = -2$

$$\frac{2}{1} \neq \frac{-5}{-4} \neq \frac{2}{-2}$$

$$2 \neq \frac{5}{4} \neq -1$$

(x) $2x - y = 3$

$2y - 4x = -6$

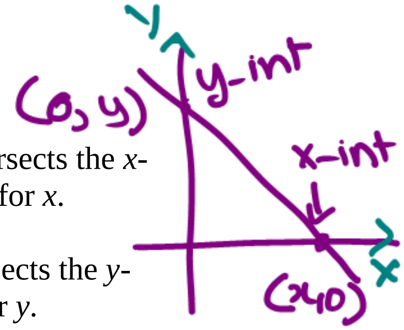
$2x - y = 3$

$-4x + 2y = -6$

$$\frac{2}{-4} \quad -\frac{1}{2} \quad \frac{3}{-6}$$

$$\underline{-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}}$$ Inf sol^n

Intercepts of Graphs



An ***x*-intercept** of a graph is the *x*-coordinate of a point where the graph intersects the *x*-axis. To find the *x*-intercept(s) of a graph set $y = 0$ in the equation and solve for *x*.

A ***y*-intercept** of a graph is the *y*-coordinate of a point where the graph intersects the *y*-axis. To find the *y*-intercept of a graph set x = 0 in the equation and solve for *y*.

Example 7: Given the following equation, find its *x*- and *y*- intercepts.

a. $3x - 4y = 0$

x-int: $y = 0$ $\quad 3x = 0$ $\quad x = 0$

y-int: $x = 0$ $\quad -4y = 0, y = 0$

b. $-2x + y = 8$

x-int! $y = 0$ $\quad -2x = 8$ $\quad x = -4$

y-int $x = 0$ $\quad y = 8$

c. $\frac{1}{14}x + \frac{5}{16}y = \frac{25}{4}$

x-int $y = 0$ $\quad \frac{1}{14}x = \frac{25}{4}$

$\cancel{14}\cdot\frac{1}{\cancel{14}}\cdot x = \frac{25}{\cancel{4}_2}\cdot\cancel{14}^{7}$

$x = \frac{175}{2}$

y-int: $x = 0$

$\frac{5}{16}y = \frac{25}{4}$

$\frac{\cancel{16}}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{16}}y = \frac{\cancel{25}^{5}}{\cancel{4}}\cdot\frac{\cancel{16}^{4}}{\cancel{5}}$

$y = 20$

Inequalities

To sketch the graph of a linear inequality, sketch the graph of the corresponding line then shade one-half plane.

Dashed

solid

- $y < mx + b$ the solution is the half-plane lying below the line $y = mx + b$
- $y \le mx + b$ the solution is the half-plane lying on or below the line $y = mx + b$
- $y > mx + b$ the solution is the half-plane lying above the line $y = mx + b$
- $y \ge mx + b$ the solution is the half-plane lying on or above the line $y = mx + b$

If the inequality is not written in slope-intercept form then you can either rewrite it so that it is and use the facts above or use a test point to determine the half plane to shade.

$4x - y = 5$

Example 8: The inequality $16x - 4y \ge 20$ is equivalent to $y \le 4x - 5$.

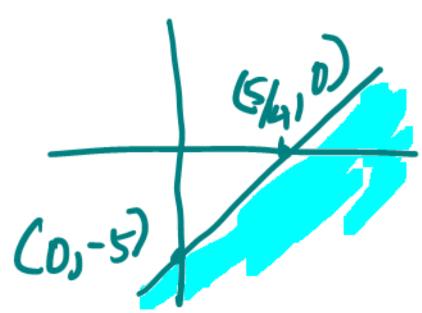
x	y
0	-5
5/4	0

$16x - 4y \ge 20$

$4x - y \ge 5$

$4x - 5 \ge y$

$y \le 4x - 5$



Example 9: Let $-2x-3y \le 4$. Which of the following describes its solution set?

$$\frac{-3y}{-3} \underset{\uparrow}{\le} \frac{4}{-3} + \frac{2x}{-3} \qquad y \ge -\frac{4}{3} - \frac{2}{3}x$$

flip

A. The solution to $-2x-3y \le 4$ is the half-plane lying above the line $-2x-3y = 4$. $>$

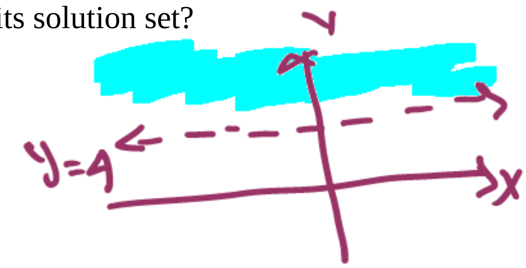
B. The solution to $-2x-3y \le 4$ is the half-plane lying above and on the line $-2x-3y = 4$. $\ge$

C. The solution to $-2x-3y \le 4$ is the half-plane lying below the line $-2x-3y = 4$.

D. The solution to $-2x-3y \le 4$ is the half-plane lying below and on the line $-2x-3y = 4$.

Example 10: Let $-16 > -4y$. Which of the following describes its solution set?

$$\frac{-16}{-4} \underset{\uparrow}{>} \frac{-4y}{-4} \qquad 4 < y$$



A. The solution to $-16 > -4y$ is the half-plane lying above the line $y = 4$.

B. The solution to $-16 > -4y$ is the half-plane lying above and on the line $y = 4$. X

C. The solution to $-16 > -4y$ is the half-plane lying below the line $y = 4$.

D. The solution to $-16 > -4y$ is the half-plane lying below and on the line $y = 4$.

Let's say that we know want to determine whether a given point is in the solution set of a linear inequality, without graphing.

We can simply take the point of interest and plug it into the linear inequality and determine if it's satisfied or not. If it is satisfied, then the point is in the solution set of the linear inequality; if not, then the point is not in its solution set.

Example 11: Determine whether each of the following statements is true or false.

A. The point (5, 0) is in the solution set of 2x + y < 10.

$2(5)+0 < 10$

$10 < 10$ False

(5,0) is not a sol^n

B. The point (-3, 8) is in the solution set of $-x + y \ge 4$

$-(-3)+(8) \ge 4$

$3+8 \ge 4$ $11 \ge 4$ True

(−3, 8) is a sol^n