12 MCQ (NO FR)
50 mins

Math 1313
Test 2 Review

How to study: Study the class notes, take your practice test, review homework problems and quizzes, and try to do as many exercises as you can from the textbook. Note that answers are provided at the back of the book to all odd numbered problems. Here I provide some examples for you. This is not a complete list, studying only these examples is not enough!

1. A piece of equipment was purchased by a company for $10,000 and is assumed to have a scrap value of $3,000 in 5 years. Assume its value is depreciated linearly.

a. What is the rate of depreciation?

$$m = \frac{3000 - 10000}{5} = \frac{-7000}{5} = -1400$$

Rate of dep. = 1400

b. Find the expression for the machine's book value in the t-th year of use ($0 \le t \le 5$).

$$V(t) = mt + \text{initial value} = -1400t + 10000$$

c. Find the value of the equipment after 3 years.

$$V(3) = -1400(3) + 10000 = \$5800$$

d. After how many years will the price be $7200?

$$7200 = -1400t + 10000$$
$$-2800 = -1400t$$
$$t = 2 \text{ years}$$

2. A bicycle manufacturer experiences monthly fixed costs of $175,000 and production costs of $75 per bicycle produced. Each bicycle sells for $125.

a. What is the cost function?

$$C(x) = 75x + 175000$$

b. What is the revenue function? $R(x) = 125x$

c. What is the profit function?

$$P(x) = R(x) - C(x)$$
$$= 125x - (75x + 175000)$$
$$= 50x - 175000$$

d. What is the break-even point?

$$P(x) = 0$$
$$50x - 175000 = 0$$
$$x = 3500$$

$$R(x) = 125(3500)$$
$$= 437500$$

B.E pt: $(3500, 437500)$

e. What is the profit (or loss) when the company produces and sells 5500 bicycles?

$$P(5500) = 50(5500) - 175000$$
$$= \$100000 \quad \text{Profit}$$

f. What is the profit (or loss) when the company produces and sells 3000 bicycles?

$$P(3000) = 50(3000) - 175000)$$
$$= -\$25000 \quad \text{Loss}$$

g. How many bicycles must the company produce and sell in order to make a profit of $62000?

$$P(x) = 50x - 175000$$
$$62000 = 50x - 175000$$
$$237000 = 50x$$
$$x = \underline{4740} \text{ bicycles}$$

3. A manufacturing company makes two types of snowboards, a standard model and deluxe model. Each standard model requires 4 labor-hours in the fabricating department and 1 labor-hour in the finishing department. Each deluxe model requires 6 labor-hours in the fabricating department and 1 labor-hour in the finishing department. The company anticipates a profit of \$50 on each standard model and \$85 on each deluxe model. If the company has at most 108 labor-hours per day available in the fabricating department and 24 labor-hours per day in the finishing department, how many of each type of snowboard should be manufactured each day to realize a maximum profit? What is the maximum profit? Set up the LPP only. Let x = number of standard models and y = number of deluxe models.

	x SM	y LM	
FB	4	6	≤ 108
FD	1	1	≤ 24
Profit	50	85	

Maximize $P = 50x + 85y$

subject to: $\begin{cases} 4x + 6y \leq 108 \\ x + y \leq 24 \end{cases}$

$$\begin{cases} 2x + 3y \leq 54 \quad (*) \\ x + y \leq 24 \end{cases}$$

$x \geq 0, \ y \geq 0 \quad [0 \leq x, 0 \leq y]$

4. You can use two types of fertilizer in your orange grove, Best Food and Natural Nutri. Each bag of Best Food contains 8 pounds of nitrogen, 4 pounds of phosphoric acid, and 2 pounds of chlorine. Each bag of Natural Nutri contains 3 pounds of nitrogen, 4 pounds of phosphoric acid and 1 pound of chlorine. You know that the grove needs at least 1,000 pounds of phosphoric acid and at most 400 pounds of chlorine. If you want to minimize the amount of nitrogen added to the grove, how many bags of each type of fertilizer should be used? How much nitrogen will be added? Set up the LPP only. Let x = number of bags of Best Food and y = number of bags of Natural Nutri.

	BF	NN	atleast (Min)
PA	4	4	≥ 1000
			atmost (Max)
Cl	2	1	≤ 400
Nit	8	3	

Minimize Nit $= 8x + 3y$

subject to: $\begin{cases} 4x + 4y \geq 1000 \\ 2x + y \leq 400 \\ x \geq 0, \; y \geq 0 \end{cases}$

$\begin{cases} x + y \geq 250 \\ 2x + y \leq 400 \\ x \geq 0, \; y \geq 0 \end{cases}$

$x + y \geq 250$

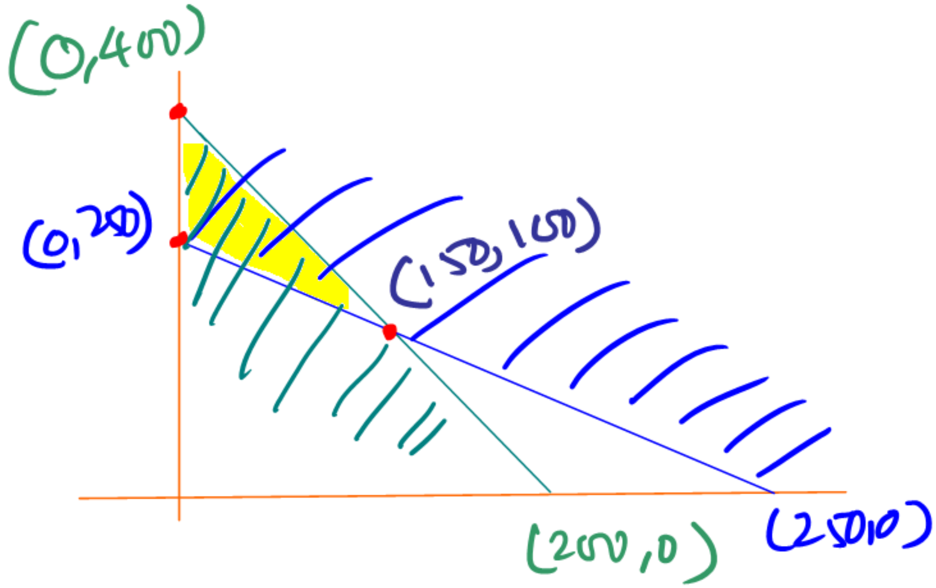
$x + y = 250$

x	y
0	250
250	0

$2x + y \leq 400$

$2x + y = 400$

x	y
0	400
200	0



$$\begin{array}{r} 2x + y = 400 \\ \underline{x + y = 250} \\ x = 150 \end{array} \qquad y = 100$$

	$8x + 3y$
(0,250)	750 ← Min
(0,400)	1200
(150,100)	1200 + 300 = 1500

optimal solⁿ:

value 750

0 # Best Foods

250 # Natural Nut.

Popper #4 1-5 → A

5. Solve the linear programming problem. Max $P = 4x + 6y$

Subject to: $\begin{cases} 2x + 5y \le 20 \\ x + y \le 7 \\ x, y \ge 0 \end{cases}$

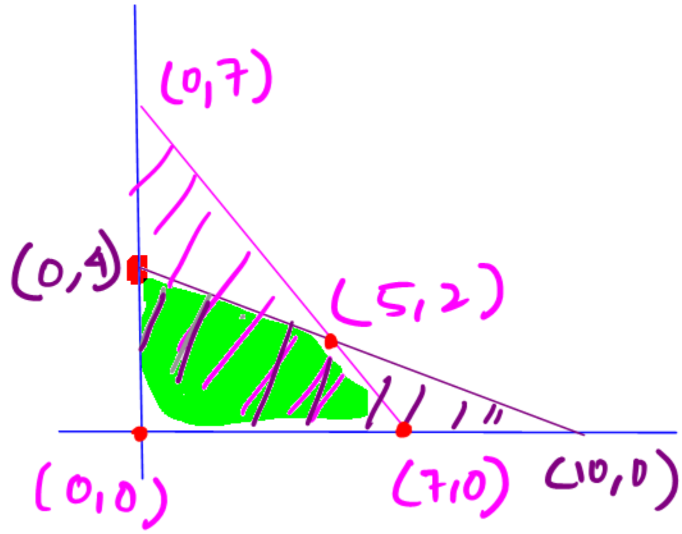
$2x+5y \le 20$

$2x+5y=20$

x	y
0	4
10	0

$x+y=7$

x	y
7	0
0	7



$2x+5y=20$
$x+y=7 \cdot \to 2$

$2x+5y=20$
$2x+2y=14$

$3y=6 \quad y=2$

$x+y=7$
$x+2=7$
$x=5$
$(5,2)$

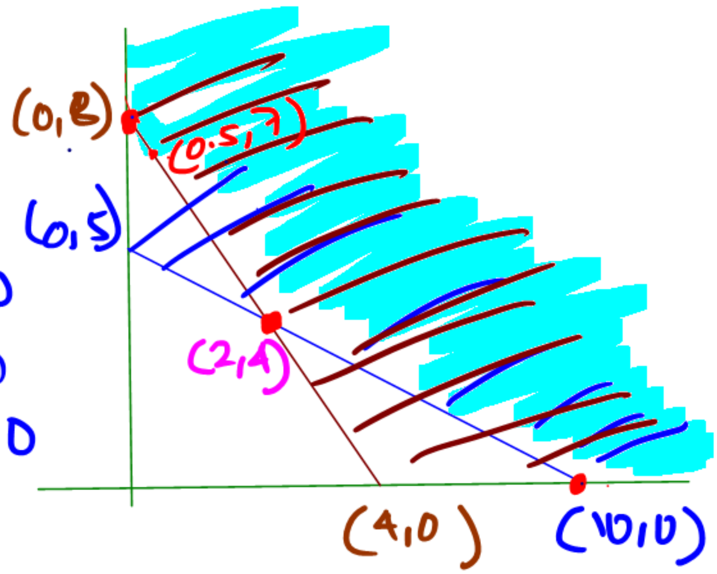
	Max $4x+6y$
(0,4)	24
(5,2)	32 ←
(7,0)	28
(0,0)	0

Opt. valu 32
Pt (5,2)

6. Use the method of corners to solve this LPP:

Minimize $Z = 5x + 2y$

Subject to: $\begin{cases} 6x + 3y \ge 24 \\ 3x + 6y \ge 30 \\ x \ge 0 \\ y \ge 0 \end{cases}$



$6x + 3y \ge 24$

$2x + y \ge 8$

$2x + y = 8$

x	y
0	8
4	0

$3x + 6y \ge 30$

$x + 2y \ge 10$

$x + 2y = 10$

x	y
0	5
10	0

$2x + y = 8$

$x + 2y = 10 \to 2$

$$\begin{array}{r} 2x + y = 8 \\ -2x - 4y = -20 \\ \hline -3y = -12 \end{array}$$

$y = 4 \quad x = 2$

	$5x + 2y$
(0,8)	$\boxed{16}$ ← min
(2,4)	18
(10,0)	50
(0.5,7)	2.5 + 14 = 16.5

Opt val 16

pt (0,8)

7. Solve the system of linear equations using the Gauss-Jordan elimination method.

a. $\begin{aligned} x + 3y + z &= 3 \\ y + 2z &= 4 \\ -9y + 2z &= -16 \end{aligned}$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & -9 & 2 & -16 \end{array}\right] \xrightarrow[R_3 = 9R_2 + R_3]{R_1 = -3R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -9 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 20 & 20 \end{array}\right]$$

$$\begin{array}{cccc} 1 & 3 & 1 & 3 \\ 0 & -3 & -6 & -12 \\ \hline 1 & 0 & -5 & -9 \end{array} \qquad \begin{array}{cccc} 0 & 9 & 18 & 36 \\ 0 & -9 & 2 & -16 \\ \hline 0 & 0 & 20 & 20 \end{array}$$

$$R_3 = \frac{1}{20} R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & -9 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[R_1 = 5R_3 + R_1]{R_2 = -2R_3 + R_2} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right] \end{array}$$

$$\begin{array}{cccc} 0 & 1 & 2 & 4 \\ 0 & 0 & -2 & -2 \\ \hline 0 & 1 & 0 & 2 \end{array}$$

$x = -4 \quad y = 2 \quad z = 1$ unique / one sol^n

b. $\begin{aligned} 2x + 3y &= 2 \\ x + 3y &= -2 \\ x - y &= 3 \end{aligned}$

$$\left[\begin{array}{cc|c} 2 & 3 & 2 \\ 1 & 3 & -2 \\ 1 & -1 & 3 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{cc|c} 1 & 3 & -2 \\ 2 & 3 & 2 \\ 1 & -1 & 3 \end{array}\right]$$

$$\begin{array}{ccc} -2 & -6 & 4 \\ 2 & 3 & 2 \\ \hline 0 & -3 & 6 \end{array}$$

$$R_2 = -2R_1 + R_2 \qquad R_3 = -R_1 + R_3$$

$$\left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & -3 & 6 \\ 0 & -4 & 5 \end{array}\right] \xrightarrow{R_2 = -\frac{1}{3} R_2} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 1 & -2 \\ 0 & -4 & 5 \end{array}\right] \xrightarrow[R_3 = 4R_2 + R_3]{R_1 = -3R_2 + R_1} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & -2 \\ 0 & 0 & -3 \end{array}\right] \end{array}$$

$x = 8$

$y = -2$

$0 = -3$ X

No sol^n

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 2 & 1 & -1 & 2 \\ 3 & 2 & -4 & 7 \end{array}\right] \xrightarrow[R_3=-3R_1+R_3]{R_2=-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & -1 & 5 & -6 \\ 0 & -1 & 5 & -5 \end{array}\right]$$

$x + y - 3z = 4$

c. $2x + y - z = 2$

$3x + 2y - 4z = 7$

$$\xrightarrow{R_3=-R_2+R_3} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & -1 & 5 & -6 \\ 0 & 0 & 0 & 1 \end{array}\right] \end{array} \xrightarrow{R_2=-R_2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 4 \\ 0 & 1 & -5 & 6 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

No sol^n

8. Indicate whether the matrix is in row-reduced form.

(a) $\left(\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 0 & 0 \end{array}\right)$ Yes

(b) $\left(\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 9 \end{array}\right)$ NO

(c) $\left(\begin{array}{ccc|c} 1 & 0 & 1 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Yes

(d) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & 1 & 2 \end{array}\right)$ NO

(e) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$ NO

(f) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Yes

(g) $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$ Yes

(h) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & 1 \end{array}\right)$ NO

9. The following augmented matrix in row-reduced form is equivalent to the augmented matrix of a certain system of linear equations. Use this result to solve the system of equations.

$x - y = 3$

$3y = 1$

$0 = -3x$

(a) $\begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left(\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & -3 \end{array}\right) \end{array}$ No sol^n

(b) $\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 9 \end{array}\right) \end{array}$

Inf many

$x + z = 2$

$y - z = 9$

$x = -5 \quad y = 2 \quad z = 3$

(c) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right)$ Unique

(d) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 3 \end{array}\right)$ No sol^n

$x = 4$

$y = -1$

$0 = 3X$