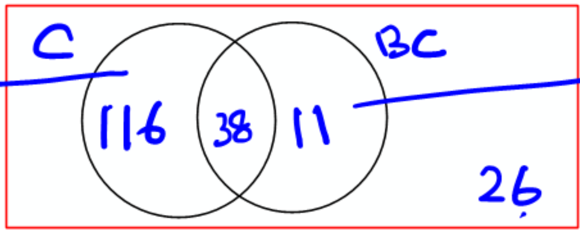
Test- 4 Review

1. A group of students were surveyed. One-hundred fifty-four are enrolled in Chemistry, 49 are enrolled in Business Calculus, 38 are enrolled in both, and 26 are not enrolled in either course.

$$\begin{array}{r} 154 \\ -\ 38 \\ \hline \end{array}$$



$49-38$

(a) How many students were surveyed?

$116+38+11+26=191$

(b) How many students are enrolled in exactly one of the two courses mentioned here?

$116+11=127$

(c) How many students are enrolled in Business Calculus or Chemistry?

$116+38+11=165$

(d) If a student is selected at random what is the probability that the student was enrolled in at most one of the subjects?

0 or 1 $\quad 116+11+26=153$

$\frac{153}{191}=0.801$

(e) If a student is selected at random what is the probability that the student was enrolled in exactly one of the subjects?

$\frac{127}{191}=0.665$

letters A – Z: 26 Digits – 0 – 9: 10

2. A license plate consists of 3 letters followed by 4 digits. How many license plates are possible if:

   (a) The 1st letter can't be O and the first digit has to be even, and no repetition of letters or digits is allowed.

No 'O' → 25 . 25 . 24 . 4 . 9 . 8 . 7 = 30240000

   (b) The letters are vowels, the first and last digits are the same and repetition is allowed for the letters but no repetition is allowed for digits.

5 5 5 10 9 8 1 = 90,000

3. A restaurant offers 6 appetizers, 4 salads, 8 entrees, and 5 desserts. In how many ways can a customer select:

   (a) One meal consisting of an appetizer, a salad, an entrée, and a dessert.

$^6C_1 \cdot {}^4C_1 \cdot {}^8C_1 \cdot {}^5C_1 = 6 \cdot 4 \cdot 8 \cdot 5 = 960$

   (b) One meal consisting of two appetizer, a salad, an entrée, and a dessert and another meal consisting of an appetizer, two salads, two entrées, and a desert.

$C(6,2) \cdot C(4,1) \cdot C(8,1) \cdot C(5,1) + C(6,1) \cdot C(4,2) \cdot C(8,2) \cdot C(5,1)$

$= 7440$

4. In how many ways can a president, vice-president and a secretary be chosen from 22 members of a club, if one person cannot hold more than one position and all 22 members are eligible for any position?

P VP S OR P(22,3)

22 21 20 9240

5. Identify whether its a permuation or combination problem and solve the following:

   (a) A committee consists of 11 people. In how many ways can a subcommittee of 4 people be chosen?

Combination C(11,4)

   (b) In how many ways can 4 people be made to seat in 11 chairs.

Permutation P(11,4)

6. (a)How arrangements are possible of the word MISSISSIPPI?

$$\frac{11!}{4!\,4!\,2!}$$

Total 11 letters

4 - S 4 - I 2 - P

(b)In how many arrangements of the word MISSISSIPPI the P's are always together?

$$\frac{10!}{4!.4!}$$

MISSISSIPI

4 - S 4 - I

7. A coin is flipped 20 times.

(a) What is the total number of outcomes?

$2^{20} = 1048576$

Probability

(b) In how many outcomes do exactly 10 tails occur?

$C(20,10) = 184756$

$$\frac{C(20,10)}{2^{20}}$$

(c) In how many outcomes do at least 18 tails occur?

18T or 19T or 20T

$C(20,18) + C(20,19) + C(20,20) = 211$

$$\frac{211}{2^{20}}$$

(d) In how many outcomes do at most 17 tails occur?

0T or 1T ... 17T OR 18T 19T 20T

1048365 $2^{20} - [C(20,18) + C(20,19) + C(20,20)]$

$$\frac{1048365}{1048576}$$

8. A crate of 17 apples contains 4 rotten apples. Seven apples are chosen at random from the crate.

(a) How many selections contain 3 rotten apples?

Probab

$$\frac{C(4,3).C(13,4)}{C(17,7)}$$

$C(4,3) . C(13,4)$

17: 4R → 3, 13G → 4

(b) How many selections contain at least 2 rotten apple?

2R, 3R 4R

$C(4,2)\,C(13,5) + C(4,3)\,C(13,4) + C(4,4)\,C(13,3)$

OR

$C(17,7) - C(13,6)\,C(4,1) - C(13,7).C(4,0)$

0 R or 1 R

17

4R 13G

(c) How many selections contain at most 1 rotten apple?

$C(4,0) \cdot C(13,7) + C(4,1)C(13,6)$

9. A judge has a jury pool of 40 people that contains 22 women and 18 men. She needs a jury of 12 people.

   (a) What is the probability that the jury contains 8 women?

   $$\frac{C(22,8) \cdot C(18,4)}{C(40,12)}$$

   40

   22 W → 8   18 M → 4

   (b) What is the probability that at most 10 men are on the jury?

   0M or 1M 2M ... 10M OR 11M . 12M

   $$1 - \frac{C(18,11) \cdot C(22,1) + C(18,12)C(22,0)}{C(40,12)} \quad 0.999$$

   (c) What is the probability that at least 11 women are on the jury?

   11W 12W

   $$\frac{C(22,11) \cdot C(18,1) + C(22,12)\, C(18,0)}{C(40,12)} \quad 0.0024$$

10. Let E and F be two events with $P(E^c) = 0.7$ and $P(F) = 0.2$ and $P(E \cup F) = 0.35$. Find:

   only E

   (a) $P(E \cap F^c)$ $= 0.15$

   (b) $P(E^c \cup F^c)$ $= P(E \cap F)^c = 1 - P(E \cap F) = 1 - 0.15 = 0.85$

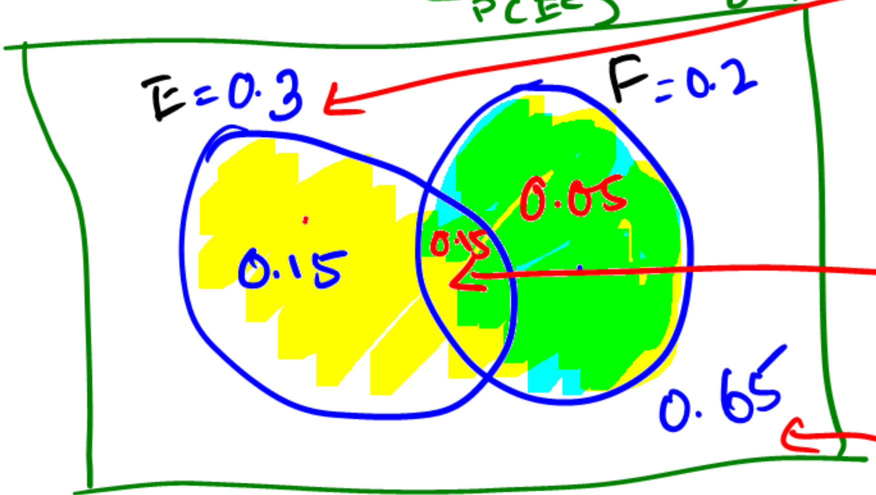
   (c) $P(E|F)$ $= \dfrac{P(E \cap F)}{P(F)} = \dfrac{0.15}{0.2} = 0.75$

   (d) $P(F|E^c)$ $= \dfrac{P(F \cap E^c)}{P(E^c)} = \dfrac{0.05}{0.7} = 0.0714$

$P(E) = 1 - P(E^c)$
$= 1 - 0.7 = 0.3$

$P(E \cup F) = 0.35$

$P(F) = 0.2$

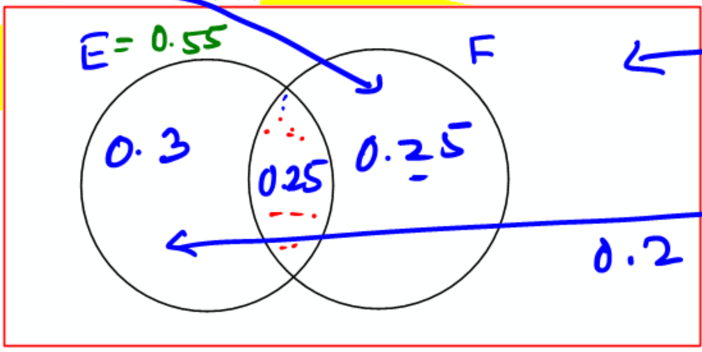


$P(E \cap F) = P(E) - P(\text{only } E)$
$= 0.3 - 0.15$

$1 - (0.15 + 0.15 + 0.05)$

$1-(0.3+0.25+0.2)=0.25$

11. Let E and F be two events with $P(E) = 0.55$ and $P(E \cap F) = 0.25$ and $P(E \cup F^c) =$ 0.75. Find:



$P(E\cup F^c) - P(E) = 0.75 - 0.55 = 0.2$

$P(\text{only } E) = P(E) - P(E\cap F) = 0.55 - 0.25 = 0.3$

(a) $P(E \cap F)^c = 0.3 + 0.25 + 0.2 = 0.75$

(b) $P(E^c \cap F^c) = P(E \cup F)^c = 0.2$

$P(F) = 0.5$

$P(F^c) = 0.5$

(c) $P(F|E) = \frac{P(F\cap E)}{P(E)} = \frac{0.25}{0.55} = 0.4545$

(d) $P(E|F^c) = \frac{P(E\cap F^c)}{P(F^c)} = \frac{P(\text{only } E)}{1-P(F)} = \frac{0.3}{1-0.5} = \frac{0.3}{0.5} = 0.$

12. Two dice are thrown and the uppermost numbers are observed. What is the probability:

(a) The sum is six.

(1,5), (5,1), (2,4), (4,2), (3,3)

$P(\text{sum } 6) = \frac{5}{36}$

(b) The second number is twice the first number.

(1,2), (2,4) (3,6) $\quad \frac{3}{36} = \frac{1}{12}$

At least twice: (1,2) (1,3) (1,4) (1,5) (1,6) (2,4) (2,5) (2,6) (3,6) $\quad \frac{9}{36} = \frac{1}{4}$

(c) The sum is at most 5, ~~given one of the number is~~ 3.

Sum

2 : (1,1)

3 : (1,2) (2,1)

4 : (1,3) (3,1) (2,2)

5 : (1,4) (4,1) (2,3) (3,2)

$\frac{10}{36}$

Sum at least 10

10 (6,4) (4,6) (5,5)

11 (6,5) (5,6)

12 (6,6)

Prob $= \frac{6}{36} = \frac{1}{6}$

13. A bag contains 7 red and 8 yellow balls. Find the probability:

(a) If two balls are drawn at random with replacement, the second ball is red.

$(R, R) \qquad (Y, R)$

$$\frac{7}{15}\cdot\frac{7}{15} + \frac{8}{15}\cdot\frac{7}{15}$$

(b) If two balls are drawn at random without replacement, the second ball is red.

$(R, R) \qquad (Y, R)$

$$\frac{7}{15}\,\frac{6}{14} + \frac{8}{15}\cdot\frac{7}{14}$$

(c) If two balls are drawn at random without replacement, the first ~~second~~ ball is red if it is known the second ball is yellow.

1st — 2nd

$\frac{7}{15}$ — R — R $\frac{6}{14}$; — Y $\frac{8}{14}$

$\frac{8}{15}$ — Y — R $\frac{7}{14}$; — Y $\frac{7}{14}$

$$P(1^{st} R \mid 2^{n} Y) = \frac{P(1^{st} R \cap 2^{nd} Y)}{P(2^{nd} Y)} = \frac{\frac{7}{15}\cdot\frac{8}{14}}{\frac{7}{15}\cdot\frac{6}{14} + \frac{8}{15}\cdot\frac{7}{14}}$$

14. Companies A, B. and C produce 10%, 40% and 50% respectively of a certain product. It has been found that 1% from A , $1\frac{1}{2}$ % from B and 2% from C are defective. One of these products is chosen at random.

0.1 — A — D 0.01 ; — ND 0.99

0.4 — B — D 0.015 ; — ND 0.985

0.5 — C — D 0.02 ; — ND 0.98

(a) Find the probability the product is defective.

$$P(D) = (0.1)(0.01) + (0.4)(0.015) + (0.5)(0.02)$$
$$= 0.017$$

$\cap$

(b) Find the probability the product is defective and that it was produced by Company C.

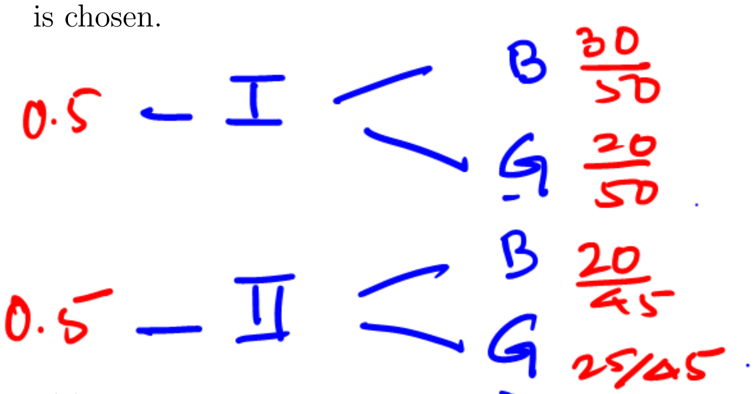
$$P(D \cap C) = (0.5)(0.02) = 0.01$$

(c) Find the probability that it produced by Company C, given it was defective.

$$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{(0.5)(0.2)}{0.017} = 0.5882$$

Part (a)

15. Urn 1 contains 30 blue and 20 green marbles. Urn 2 contains 20 blue and 25 green marbles. An urn is chosen at random with equally likely probability, then a marble is chosen.



(a) What is the probability that the marble chosen was green?

$$P(G) = (0.5)\left(\frac{20}{50}\right) + (0.5)\left(\frac{25}{45}\right) = 0.4778$$

(b) What is the probability the marble was green, given that it was taken from Urn 2.

$$P(G|II) = \frac{25}{45}$$

$$P(G|II) = \frac{P(G \cap II)}{P(II)} = \frac{(0.5)(25/45)}{0.5} = \frac{25}{45}$$

(c) What is the probability Urn 1 was chosen, if the marble was green?

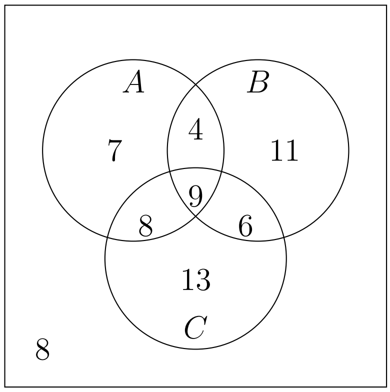
$$P(I|G) = \frac{P(I \cap G)}{P(G)} = \frac{(0.5)\left(\frac{20}{50}\right)}{0.4778} = 0.4186$$

$$(*)\ P(I|B) = \frac{P(I \cap B)}{P(B)} = \frac{(0.5)\left(\frac{30}{50}\right)}{1 - P(G)}$$

$$P(B) = 1 - P(G) \text{ OR } P(B) = (0.5)\left(\frac{30}{50}\right) + (0.5)\left(\frac{20}{45}\right)$$

Popper 11/11 (#26)
1–5: B

16. Consider the give Venn diagram. The numbers represents the number of elements in following set region.Find:



(a) $n(C \cup A^c)$ $= 11+9+8+6+13+8$

(b) $n(B^c \cap A)^c$ $= n(B \cup A^c) = 11+4+9+6+13+8$

(c) $n(C \cup (A \cap B)^c)$ $= 7+11+9+8+6+13+8$

(d) $n(A^c \cap (B^c \cup C))$ $= 6+13+8$

(e) $n(B \cup (A \cup C^c)^c)$ $= n(B \cup (A^c \cap C)) = 11+4+9+6+13$

17. *Lets try a variation of our Panda express problem with Chipotle*
You wish to have food form Chipotle. You ordering online for delivery has a probability of 0.6. If you order online, you are likely to order burrito bowl with probability 0.75. If you go in store you are likely not to order burrito bowl with probability 0.45. What is the probability of:

0.6 — Online — BB 0.75
— NBB 0.25 (1−0.75).

(1−0.6) 0.4 — Instore — BC 0.55
— NBB 0.45.

(a) Not ordering burrito bowl.

$P(NBB) = (0.6)(0.25) + (0.4)(0.45) = 0.33$

(b) Ordering burrito bowl if you have gone to the store.

$P(BB \mid Instore) = 0.55$ | * Order BB & Instore $= (0.4)(0.55)$

(c) Going to store if you have not ordered burrito bowl.

$$P(Instore \mid NBB) = \frac{P(Instore \cap NBB)}{P(NBB)} = \frac{(0.4)(0.45)}{(0.33)} = 0.5455$$