Depriciation = decrease in value

scrap value = remaining value

$$\begin{array}{r} 250000 \\ 10000 \\ \hline 240000 \end{array}$$

Section 1.5A
Linear Functions and Math Models

Simple Depreciation

Example 1: In 2000, the B&C Company installed a new machine in one of its factories at a cost of $250,000. The machine is depreciated linearly over 10 years with a scrap value of $10,000.

a. Find the rate of depreciation for this machine.

(time, value)
(0, 250000)
(10, 10000)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{scrap.v} - \text{int val}}{\text{time}} = \frac{10000 - 250000}{10}$$

$$= -24{,}000$$

b. Find an expression for the machine's book value in the *t*-th year of use ($0 \le t \le 10$).

$v(t) = mt + \text{initial value}$ $\qquad$ $V(t) = -24000t + 250000$

c. Find the machine's book value at the end of the 7^{th} year.

$$v(7) = -24000 \cdot 7 + 250000 = \$82{,}000$$

Example 2: A company car has an original value of $35,250 and it will be depreciated linearly over 5 years with a scrap value of $7,000.

a. Find the rate of depreciation for this car.

(time, value)
(0, 35250)
(5, 7000)

$$m = \frac{(7000 - 35250)}{5} = -5650$$

$7000 - 35250/5$ X

b. Find an expression for the car's book value in the *t*-th year of use ($0 \le t \le 5$).

$$v(t) = mt + \text{initial value}$$
$$= -5650t + 35250$$

(c) what the value after 3 years?

$$v(3) = -5650(3) + 35250 = \$18300$$

Linear Cost, Revenue and Profit Functions

Let x be the number of units of a product manufactured or sold at a company then:

The **cost function**, $C(x)$, is the total cost of manufacturing x units of the product.

Fixed costs are costs that remain more or less constant regardless of the company's activity level.

Example: rental fees and executive salaries

Variable costs are costs that vary with production or sales.

Example: wages and costs for raw material

The **revenue function**, $R(x)$, is the total revenue realized from the sale of x units of the product.

The **profit function**, $P(x)$, is the total profit realized from manufacturing and selling x units of the product.

Formulas

Suppose a company has fixed cost of F dollars, production cost of c dollars per unit and selling price of s dollars per unit then

$x = 10$
$s = \$7$
$R(x) = \$70$
$P = 70 - 150$
$= -80$

$x = 10$
$c = \$5$
$F\ \$100$
$C(x) = 10 \cdot 5 + 100$
$= \$150$

$C(x) = cx + F$
$R(x) = sx$
$P(x) = R(x) - C(x) = (s - c)x - F$

where x is the number of units of the product produced and sold.

Example 2: A manufacturer has a monthly fixed cost of \$100,000 and a production cost of \$14 for each unit produced. The product sells for \$20 per unit.

a. Find the cost, revenue and profit functions.

$C(x) = 14x + 100000$

$R(x) = 20x$

$P(x) = R(x) - C(x) = 20x - (14x + 100000) = 6x - 100000$

b. Compute the profit (loss) corresponding to production levels of 15,000 units and 27,500 units.

$P(15000) = 6(15000) - 100000 = -\$10{,}000$ (Loss)

$P(27500) = 6(27500) - 100000 = \$65{,}000$ (Profit)

Example 3: A company that manufactures motorcycle helmets has monthly fixed costs of $55,000 and ~~monthly~~ cost of $21 per helmet. The selling price for each unit is $41.
a. How many helmets must the company produce and sell if they wish to make a profit of $50,000?

$C(x) = 21x + 55000$

$R(x) = 41x$

$P(x) = 41x - (21x + 55000) = 20x - 55000$

$20x - 55000 = 50000$

$20x = 105000 \quad x = 5250$ helmets

b. What is the profit (loss) if they produce and sell 3500 helmets?

$P(3500) = 20(3500) - 55000$

$= \$15{,}000$ (Profit)