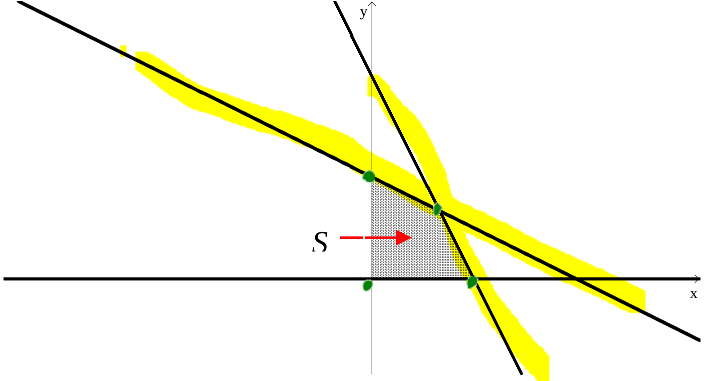
Section 2.1
Solving Linear Programming Problems

A function subject to a system of constraints to be optimized (maximized or minimized) is called an **objective function.**

A system of equalities or inequalities to which an objective function is subject to are called **constraints.**

An objective function subject to a system of constraints is called a **linear programming problem.**

Consider the following figure which is associated with a system of linear inequalities:



The set *S* is called a **feasible set.** Each point in *S* is a candidate for the solution of the problem and is called a **feasible solution.**

The point(s) in *S* that optimizes (maximizes or minimizes) the objective function is called the **optimal solution.**

Theorem 1 (in book) Linear Programming

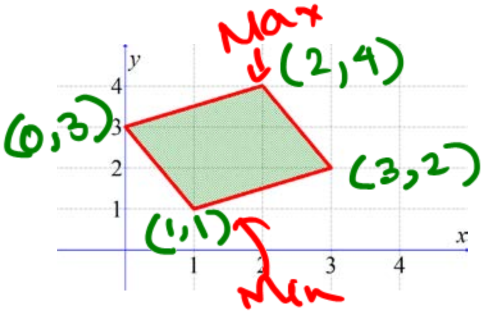
If a linear programming problem has a solution, then it must occur at a vertex, or corner point of the feasible set S associated with the problem. Furthermore, if the objective function P is optimized at two adjacent vertices of S, then it is optimized at every point on the line segment joining these vertices, in which case there are infinitely many solutions to the problem.

To solve a linear programming problem we use the method of corners.

The Method of Corners

1. Graph the feasible set (graph the system of constraints).
2. Find the coordinates of all corner points (vertices) of the feasible set.
3. Evaluate the objective function at each corner points.
4. Find the vertex that renders the objective function a maximum (minimum).

Example 1: Given the following feasible set and objective function, $F = 7x + 8y$, find the maximum and minimum value of the objective function.



	$F = 7x + 8y$
(0, 3)	7(0) + 8(3) = 24
(2, 4)	7(2) + 8(4) = 46 ← Max
(3, 2)	7(3) + 8(2) = 37
(1, 1)	7(1) + 8(1) = 15 ← Min

Example 2: Minimize C = 5x + 8y

Subject to
$x + y \geq 5$
$x \geq 0$
$y \geq 0$
$x \leq 1$
$y \leq 5$

Step 1: Graph the feasible set.

(1) $x + y = 5$ (2) $x = 0$ (3) $y = 0$ (4) $x = 1$ (5) $y = 5$

x	y
0	5
5	0

Step 2: Find the corner points of the feasible set.

(1,4) 37

Step 3: Find where the optimal solution occurs and the optimal value.

Corner Points	Min C = 5x + 8y
(0,5)	5(0) + 8(5) = 40
(1,5)	5(1) + 8(5) = 45
(1,4)	5(1) + 8(4) = 37 ← Min

Example 3: Maximize $Z = 6x + 2y$

Subject to $6x + 3y \le 24$

$3x + 6y \le 30$

$x \ge 0$

$y \ge 0$

$6x + 3y = 24$
$3x + 6y = 30$
use sub or elim to find x & y

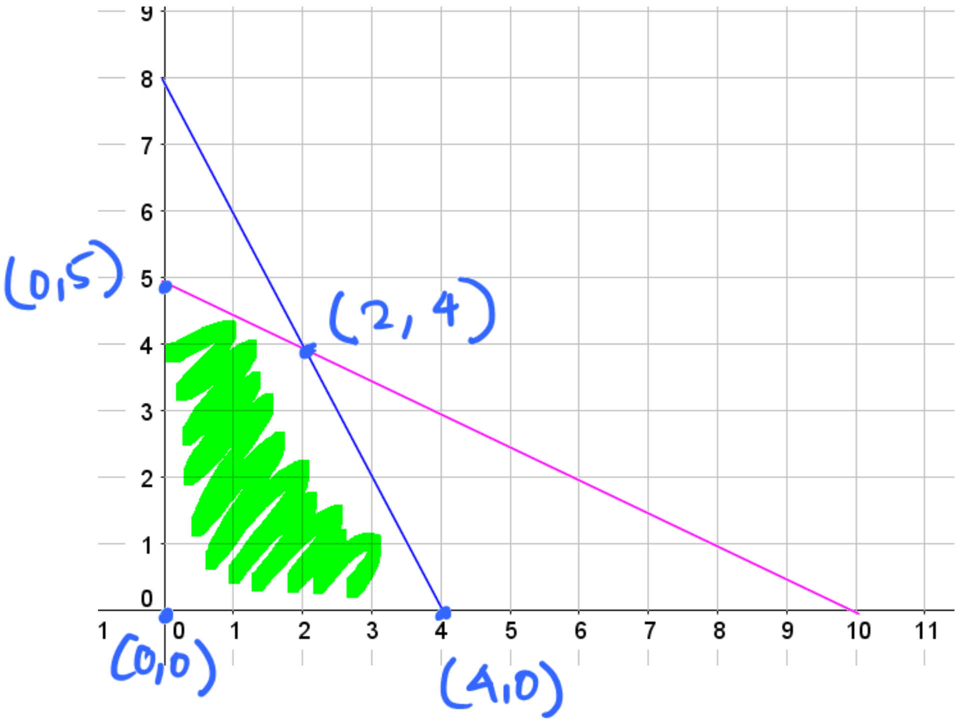
Step 1: Graph the feasible set.

(1) $6x + 3y = 24$

x	y
0	8
4	0

(2) $3x + 6y = 30$

x	y
0	5
10	0



Step 2: Find the corner points of the feasible set.

$6x + 3y = 24$

$3x + 6y = 30$

Step 3: Find where the optimal solution occurs and the optimal value.

Corner Points	Max $Z = 6x + 2y$
(0,0)	0
(4,0)	6(4) + 2(0) = 24 ← Max
(2,4)	6(2) + 2(4) = 20
(0,5)	6(0) + 2(5) = 10

Example 4: Minimize $R = 3x + 4y$

Subject to $x + y \geq 20$

$x + 3y \geq 36$

$x \geq 0$

$y \geq 0$

Step 1: Graph the feasible set.

(1) $x + y = 20$

(2) $x + 3y = 36$

x	y
36	0
0	12

x	y
0	20
20	0

$x+y=20$

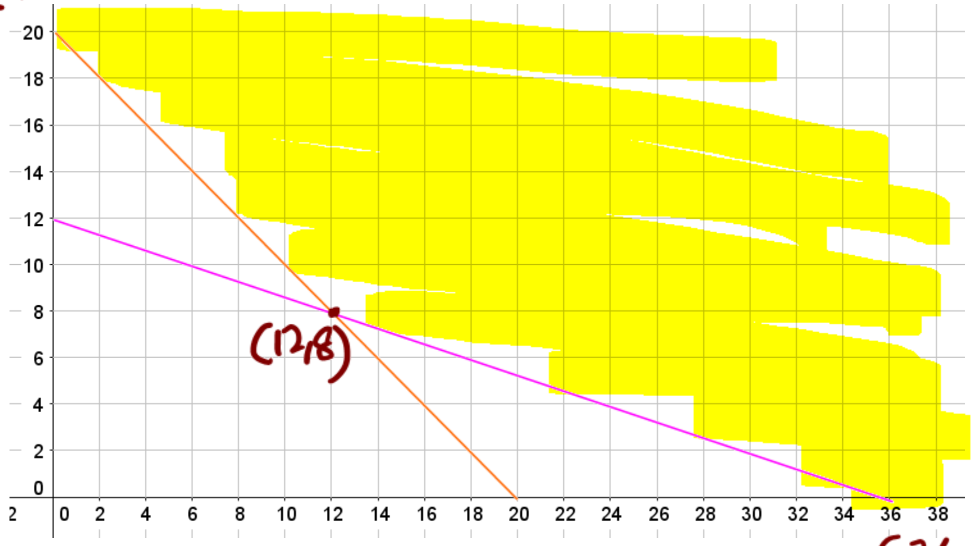
$-\underline{x+3y=36}$

$-2y=-16$

$y=8$

$x=12$

(0,20)



(36,0)

Step 2: Find the corner points of the feasible set.

$x + y = 20$

$x + 3y = 36$

(12,8)

(36,0)

(0,20)

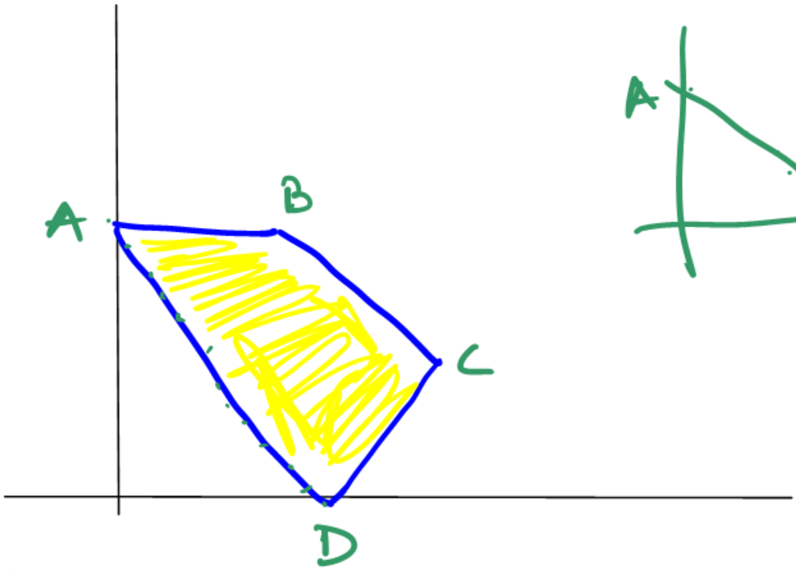
What if Max?

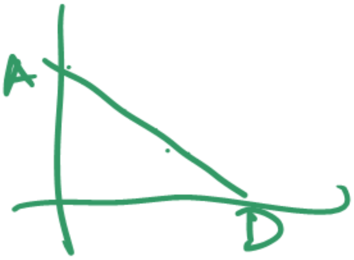
Step 3: Find where the optimal solution occurs and the optimal value.

Corner Points	Min $R = 3x + 4y$
→ (12,8)	$3(12)+4(8)=68$ ← Min
(36,0)	$3(36) = 108$
(0,20)	$4(20) = 80$
(14,8)	$42 + 32 = 74$
(10,10)	$30 + 40 = 70$
(10,12)	$30 + 48 = 78$

	Max $3x+4y$
(36,0)	108
(38,0)	$3(38)=114$
(0,30)	120

No sol^n





Max ?

Pt		value
A	→	35
B	→	25
C	→	20
D	→	35

Infitnely many sol^n
all points on the line $\overline{AD}$
will give the optimal value

$x + y = 5$