

Section 3.1 Matrices

A **matrix** (plural matrices) is a rectangular array of numbers, letters, symbols, or algebraic expressions that are arranged in rows and columns.

Matrices aid in solving linear systems of equations but before we learn how, we'll need to know some basic matrix definitions.

The entry in the i th row and j th column is denoted by a_{ij} . The expressions that make up the matrix are called **entries** or **elements** of the matrix.

A matrix with m rows and n columns has **size** or **dimension** $m \times n$.

Example 1: Given $A = \begin{pmatrix} 2 & 7 & -7 \\ -5 & 3 & 9 \\ 0 & -10 & 20 \\ 1 & -3 & -11 \end{pmatrix}$,

- a. what is the dimension of A ? 4×3
- b. identify a_{33} . $= 20$
- c. identify a_{12} . $= 7$

$$a_{42} = -3$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (3 \times 1)$$

A matrix with only one column or one row is called a **column matrix** (or **column vector**) or **row matrix** (or **row vector**), respectively.

$$(1 \ 2 \ 3) \quad (1 \times 3)$$

Matrix Representation

In order to write a system of linear equations in matrix form, first make sure the like variables occur in the same column. Then we'll leave out the variables of the system and simply use the coefficients and constants to write the matrix form.

$$2x + 4y + 6z = 22$$

Example: Given $3x + 8y + 5z = 27$.

$$-x + y + 2z = 2$$

The coefficient matrix contains only the coefficients of the system

Coefficient Matrix: $\begin{pmatrix} x & y & z \\ 2 & 4 & 6 \\ 3 & 8 & 5 \\ -1 & 1 & 2 \end{pmatrix}$.

The constant matrix contains only the constants of the system.

$$\text{Constant Matrix: } \begin{pmatrix} 22 \\ 27 \\ 2 \end{pmatrix}$$

The augmented matrix contains both the coefficients and constants of the system. The vertical line is used to separate the coefficients from the constants.

$$\text{Augmented Matrix: } \left(\begin{array}{ccc|c} 2 & 4 & 6 & 22 \\ 3 & 8 & 5 & 27 \\ -1 & 1 & 2 & 2 \end{array} \right)$$

Example 2: Given the following system of equations, write it in augmented matrix form.

$$\begin{aligned} 2x - 4y + z &= 6 \\ -3x + 6y - 5z &= -1 \\ x - 3y + 7z &= 0 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 2 & -4 & 1 & 6 \\ -3 & 6 & -5 & -1 \\ 1 & -3 & 7 & 0 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 2 & -3 & 1 & 6 \\ -4 & 6 & -3 & -1 \\ 1 & -5 & 7 & 0 \end{array} \right) \times$$

$$\textcircled{x} \quad \begin{aligned} 2x + 3y + z &= 1 \\ -5z &= 2 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 2 & 3 & 1 & 1 \\ 0 & 0 & -5 & 2 \end{array} \right)$$