Section 3.3: Matrix Operations

Addition and Subtraction of Matrices

If A and B are two matrices of the same size,

1. A + B is the matrix obtained by adding the corresponding entries in the two matrices.

2. A – B is the matrix obtained by subtracting the corresponding entries in B from A.

Laws for Matrix Addition

If A, B, and C are matrices of the same dimension, then

1. A + B = B + A

2. (A + B) + C = A + (B + C)

Example 1: Refer to the following matrices: If possible,

$$A = \begin{bmatrix} 8 & -3 & 1 \\ 0 & -9 & -4 \\ 9 & 6 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} -5 & 4 & -1 \\ 8 & 4 & 8 \\ 10 & 15 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 10 & -8 & 3 \\ 5 & -4 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 1 & 3 \\ 8 & 5 & 1 \end{bmatrix}$$

3x3 3x3 2x3 2x3

a. compute A – B

$$\begin{bmatrix} 8 & -3 & 1 \\ 0 & -9 & -4 \\ 9 & 6 & 7 \end{bmatrix} - \begin{bmatrix} -5 & 4 & -1 \\ 8 & 4 & 8 \\ 10 & 15 & -2 \end{bmatrix} = \begin{bmatrix} 8-(-5) & -3-4 & 1-(-1) \\ 0-8 & -9-4 & -4-8 \\ 9-10 & 6-15 & 7-(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -7 & 2 \\ -8 & -13 & -12 \\ -1 & -9 & 9 \end{bmatrix} \quad 3x3$$

b. compute B + C.

Not possible

c. compute D + C.

$$\begin{bmatrix} 10 & -8 & 3 \\ 5 & -4 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 1 & 3 \\ 8 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 14 & -7 & 6 \\ 13 & 1 & 3 \end{bmatrix}_{2x3}$$

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Scalar Multiplication

A **scalar** is a real number.

Scalar multiplication is the product of a scalar and a matrix. To perform scalar multiplication, each element in the matrix is multiplied by the scalar; hence, it "scales" the elements in the matrix

Example 2: Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 4 \\ -7 & 9 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 2 & 3 \\ -6 & -9 & 1 \end{pmatrix}$ find, if possible,

a. -3C

$$-3\begin{pmatrix} 1 & 2 & 3 \\ -6 & -9 & 1 \end{pmatrix} = \begin{pmatrix} -3 & -6 & -9 \\ 18 & 27 & -3 \end{pmatrix}$$

b. -2B – A

$$-2\begin{pmatrix} -1 & 4 \\ -7 & 9 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & -8 \\ 14 & -18 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -10 \\ 11 & -22 \end{pmatrix}$$

c. 3B + 2C

2x2 2x3

Not possible

Transpose of a Matrix

If A is an m x n matrix with elements a_{ij}, then the **transpose** of A is the n x m matrix A^T with elements a_{ji}.

a_{12} a_{21}

$A = n \times m$

$$A = \begin{bmatrix} 2 & 5 & 50 \\ 1 & 3 & 27 \\ 16 & 45 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 1 & 16 \\ 5 & 3 & 45 \\ 50 & 27 & 1 \end{bmatrix}$$

$A^T = m \times n$

Example 3: Given the following matrices, find their transpose.

a. $B = \begin{pmatrix} -3 & 0 & 6 \\ 10 & 100 & 3 \end{pmatrix}$

2x3

$$B^T = \begin{pmatrix} -3 & 10 \\ 0 & 100 \\ 6 & 3 \end{pmatrix}$$

(3x2)

b. $D = \begin{pmatrix} 0 \\ -4 \\ 11 \\ -3 \end{pmatrix}$ (4×1)

$D^T = \begin{pmatrix} 0 & -4 & 11 & -3 \end{pmatrix}$ (1×4)

A **square matrix** is a matrix having the same number of rows as columns.

Ex: $\begin{pmatrix} 3 & 9 \\ 4 & 1 \end{pmatrix}$ $A = \underline{m} \times \underline{m}$

Equality of Matrices

Two matrices are equal if they have the same dimension and their corresponding entries are equal.

Example 4: Solve the following matrix equation for w, x, y, and z.

$$\begin{bmatrix} w+6 & x \\ y-2 & z \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}$$

$w + 6 = -2$ $x = 0$

$y - 2 = 1$ $z = 4$

$w = -8$ $x = 0$

$y = 3$ $z = 4$

Example 5: Solve for the variables in the matrix equation.

$$-\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix} + 9\begin{bmatrix} u-6 & 2z+5 \\ y & -\frac{1}{3} \end{bmatrix} = -2\begin{bmatrix} 3 & -8 \\ 1 & v \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ -4 & -3 \end{bmatrix} + \begin{bmatrix} 9u-54 & 18z+45 \\ 9y & -3 \end{bmatrix} = \begin{bmatrix} -6 & 16 \\ -2 & -2v \end{bmatrix}$$

$$\begin{bmatrix} 9u-55 & 18z+47 \\ 9y-4 & -6 \end{bmatrix} = \begin{bmatrix} -6 & 16 \\ -2 & -2v \end{bmatrix}$$

$9u - 55 = -6$ $18z + 47 = 16$ $9y - 4 = -2$ $-6 = -2v$

$9u = 49$ $18z = -31$ $9y = 2$ $3 = v$

$u = 49/9$ $z = -31/18$ $y = 2/9$