Section 3.5 The Inverse of a Square Matrix

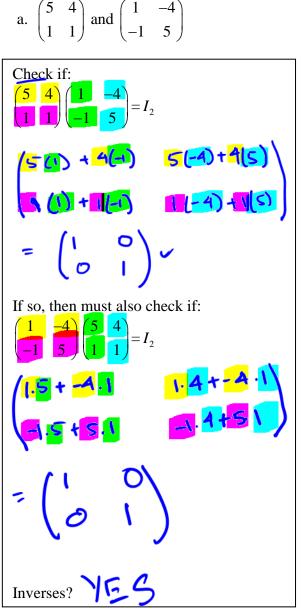
In this section we discuss a procedure for finding the inverse of a matrix and show how the inverse can be used to help us solve a system of linear equations.

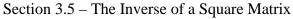
Let A be a square matrix of size n. A square matrix A^{-1} of size n such that $AA^{-1} = A^{-1}A = I_n$ is called the **inverse of A**.

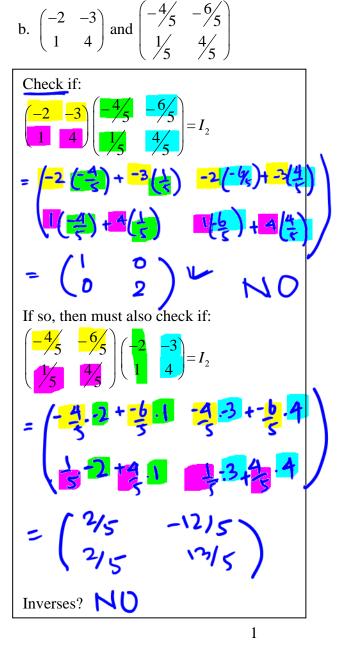
2.1 = 2 $2 \cdot \frac{1}{2} = 1$

Note: Not every square matrix has an inverse. A matrix with no inverse is called **singular**.

Example 1: Determine whether the following pairs of matrices are inverses of each other.







How to Find the Inverse of a Square Matrix

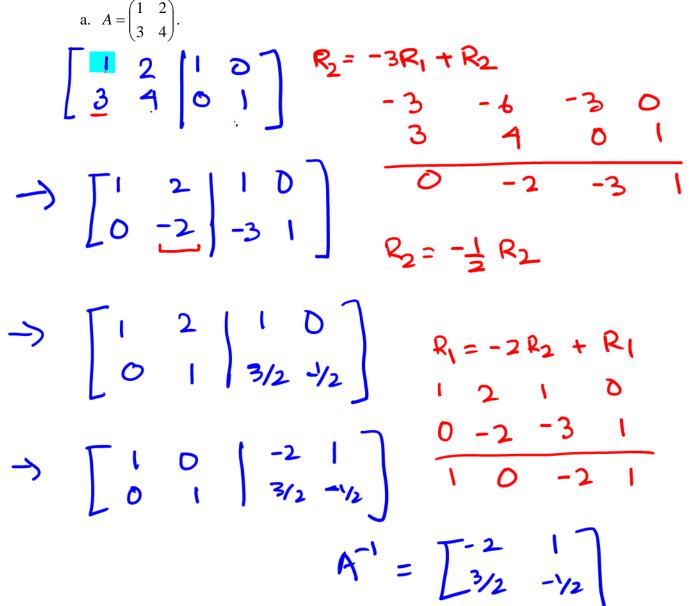
1. Adjoin the square matrix A with the identity matrix of the same size, [A | I].

2. Use the Gauss-Jordan elimination method to reduce [A | I] to the form [I | B], if possible.

The matrix *B* is the inverse of the matrix *A*. It may be verified by checking $AB = BA = I_n$.

If in the process of reducing [A | I] to [I | B], there is a row of zeros to the left of the vertical line then the matrix does not have an inverse.

Example 2: Find the inverse of each matrix, if possible.



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$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{2} = -2R_{1} + R_{2} \\ R_{3} = -3R_{1} + R_{3} \\ R_{3} = -R_{2} + R_{3} \\ R_{3} = R_{3} + R_{3} \\ R_{3} = -R_{2} + R_{3} \\ R_{3} = R_{3} + R_{3} \\ R_{4} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \\ R_{3} = R_{3} + R_{3} \\ R_{4} = R_{3} + R_{4} \\ R_{4} = R_{3} + R_{4} \\ R_{4} = R_{4} + R_{4} \\ R_$$

Section 3.5 – The Inverse of a Square Matrix

Formula for the Inverse of a 2X2 Matrix

Let
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
. Suppose $D = ad - bc$ is not equal to zero. Then. A^{-1} exists and is
given by $A^{-1} = \frac{1}{D} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ Tr $D = D$ then there is no
inverse
He matrix is singular
Example 3: Find the inverse of the following matrix.
a. $E = \begin{pmatrix} -5 & 10 \\ -2 & 7 \end{pmatrix}$ b. $F = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$
 $D = -6 \cdot 2 - 3 \cdot 4$
 $= -12 + 12 = 0$
No inverse $f = -15 = 10$
 $2 -5 = (-7/15 - 2/3)$
No inverse $f = -15 = (-7/15 - 2/3)$

The use of inverses to solve systems of equations is advantageous when we are required to solve more than one system of equations AX = B, involving the same coefficient matrix, A, and different matrices of constants, B.

The steps are:

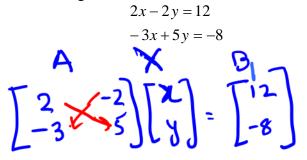
1. Write the system of equations in matrix form, i.e., AX= B. *A is the coefficient matrix*, *X is the variable matrix and B is the constant matrix*.

- 2. Find the inverse of the coefficient matrix, A.
- 3. Multiply both sides by A^{-1} .
- 4. State the answer.

AX = B $A^{T}AX = A^{T}B$ $X = A^{T}B$

2x - 2y = 0-3x + 5y = 4

Example 4: Write each system of equations as a matrix equation, then solve the system using the inverse of the coefficient matrix.



$$A = \begin{pmatrix} 2 & -2 \\ -3 & 5 \end{pmatrix}$$

Now find the inverse of the coefficient matrix.

$$D = ad - bc = 2 \cdot 5 - (-2)(-3) = 10 - 6 = 4 \neq 0$$

$$A^{-1} = \frac{1}{4} \begin{pmatrix} 5 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 5/4 & 1/2 \\ 3/4 & \sqrt{2} \end{pmatrix}$$

$$AX = B_{1}$$

$$X = A^{-1}B_{1}$$

$$\begin{bmatrix} X \\ Y \\ Y \end{bmatrix} = \begin{pmatrix} 5/4 & 1/2 \\ 3/4 & \sqrt{2} \end{pmatrix} \begin{pmatrix} 1/2 \\ 1/2 \\ 1/2 \end{pmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Y \end{bmatrix} = \begin{pmatrix} 5/4 & 1/2 \\ 3/4 & \sqrt{2} \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1/2 \\ 0 \\$$

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Anxn Bnxn AB = nxn BAnxn AB = BA in general De matrix multiplication is not commutative ingeneral BUT Matrix addition is commutative A+B = B+A $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} AB \neq BA$ $BA = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} J$