Section 4.1
Simple Interest, Future Value and Present Value

Interest that is computed on the original principal only is called **simple interest.**

Formula: $I = Prt$

where P = principal (original)
r = rate
t = time (in years)

The sum of the principal and interest after *t* years is called the **accumulated amount.**

Formula: $F = P(1+rt) = P + Prt = P + I$

Example 1: Find the simple interest on a $1,000 investment made for 3 years at an interest rate of 5% per year. What is the accumulated amount? F ?

$P = 1000 \quad r = 0.05 \quad t = 3$

$I = Prt = 1000(0.05)(3) = \$150$

$F = 1000 + 150 = \$1150$

Example 2: Find the simple interest rate at which $1,000 will grow to $1,050 in 9 months. r ?

$P = 1000 \quad F = 1050$

$I = F - P = 1050 - 1000 = 50$

$I = Prt \qquad 50 = 1000 \cdot r \cdot \frac{9}{12}$

$r = \frac{50 \cdot 12}{1000 \cdot 9} = \frac{3}{45}$

$= 0.0667$

$= 6.67\%$

Earned interest that is periodically added to the principal and thereafter itself earns interest at the same rate is called **compound interest.**

Future Value with compound interest Formula:

$$F = P(1+i)^n \qquad \text{where } i = \frac{r}{m} \text{ and } n = mt.$$

F stands for the **Future Value** or the accumulated amount at the end of *n* conversion periods. A **conversion period** refers to the interval of time between successive interest calculations.

P stands for the **Present Value** or principal.
r stands for the interest rate per year.
m stands for the number of conversion periods per year.
t stands for time (in years).

$m = \begin{cases} 1 & \text{annually} \\ 2 & \text{semi-anually} \\ 4 & \text{quarterly} \\ 12 & \text{monthly} \end{cases}$

Example 3: Find the accumulated amount after 5 years if $1700 is invested at 6.25% per year compounded quarterly.

F?

F.V with CI

$P = 1700$, $t = 5$ yrs, $r = 0.0625$

$m = 4$, $i = \frac{0.0625}{4}$, $n = 4.5 = 20$

$F = 1700\left(1 + \frac{0.0625}{4}\right)^{20}$

$= \$2318.02$

Recall $F = P(1 + i)^n$ and P = Present Value. Solving the Future Value formula for P we obtain the

Present Value with compound interest formula:

$$P = F(1+i)^{-n}$$

Example 4: A newborn child receives a $5,000 gift towards a college education from her grandparents. How much will the $5,000 be worth in 17 years if it is invested at 9% per year compounded quarterly?

F.V with CI.

$P = 5000$, $t = 17$, $r = 0.09$, $m = 4$

$i = \frac{0.09}{4}$, $n = 4.17 = 68$

$F = P(1+i)^n$

$= 5000\left(1 + \frac{0.09}{4}\right)^{68}$

$= \$22702.60$

Example 5: In a certain area of a local town, housing costs have been increasing at 6% per year compounded annually for the past 4 years. A house currently worth $200,000 would have had what value 4 years ago?

P.V. with C.I

$P = ?$

$F = 200000$

$t = 4$

$m = 1$

$r = 0.06$

$i = \frac{0.06}{1} = 0.06$

$n = 4$

$P = F(1+i)^{-n}$

$= 200000(1+0.06)^{-4}$

$= \$158418.73$

Example 6: An Individual Retirement Account (IRA) has $20,000 in it and the owner decides not to add any more money to the account other than interest earned at 8% per year compounded monthly. How much will be in the account 35 years from now when the owner reaches retirement age?

F.V. with C.I

$F = ?$

$P = 20{,}000$

$r = 0.08$

$t = 35$

$m = 12$

$i = \frac{0.08}{12}$

$n = 12(35) = 420$

$F = P(1+i)^{n}$

$= 20000\left(1+\frac{0.08}{12}\right)^{420}$

$= \$325851$

Example 7: Kaylin's son will be leaving to an out-of-state private university this year. Twenty years ago she set up an account to help pay for his college tuition. She pays out the total amount earned, which is $25, 678.90. How much did she originally invest in this account at the rate of 7% per year compounded monthly?

P.V with C I

$P = ?$

$F = 25678.90$

$r = 0.07$

$t = 20$

$m = 12$

$i = \frac{0.07}{12}$

$n = 12(20) = 240$

$P = F(1+i)^{-n}$

$= 25678.90\left(1+\frac{0.07}{12}\right)^{-240}$

$= \$6358.15$