Section 4.3
Amortization and Sinking Funds

To **Amortize** means to pay off a debt by installment payments.

Amortization Formula

The periodic payment E on a loan of P dollars to be amortized over n periods with interest charged at the rate of i per period is

$$E = \frac{Pi}{1-(1+i)^{-n}}$$

Sinking Fund

A fund that is set up for a specific purpose at some future date is called a **sinking fund.**

Sinking Fund Formula

The periodic payment E required to accumulate a sum of F dollars over n periods with interest charged at the rate of i per period is

$$E = \frac{iF}{(1+i)^n - 1}$$

Example 1:   Kelly wishes to buy a car that costs $32,998. The car dealer tells her that they can finance the car at 6.25% per year compounded monthly for 60 months. She decides to secure the loan from the dealer. How much will her monthly payments be?

Amortization
$$E = \frac{Pi}{1-(1+i)^{-n}}$$

Sinking Fund
$$E = \frac{iF}{(1+i)^n - 1}$$

$$i = \frac{0.0625}{12}$$

$$n = 60$$

$$E = \frac{32998\left(\frac{0.0625}{12}\right)}{1-\left(1+\frac{0.0625}{12}\right)^{-60}} = \$641.79$$

Example 2: A person would like to have $200,000 in an account for retirement 15 years from now. How much should be deposited quarterly in an account paying 6% per year compounded quarterly to obtain this amount?

Amortization

$$E = \frac{Pi}{1-(1+i)^{-n}}$$

Sinking Fund

$$E = \frac{iF}{(1+i)^n - 1}$$

$$i = \frac{0.06}{4}$$

$$n = 4(15) = 60$$

$$E = \frac{\left(\frac{0.06}{4}\right) 200000}{\left(1 + \frac{0.06}{4}\right)^{60} - 1} = \$2078.69$$

Example 3: The cash price of the sailboat you wish to purchase is $16,000. You pay 15% down and secure a loan for the remaining balance. How much are your monthly payments if 18% per year compounded monthly is charged over a period of 6 years?

Amortization

$$E = \frac{Pi}{1-(1+i)^{-n}}$$

Sinking Fund

$$E = \frac{iF}{(1+i)^n - 1}$$

$$i = \frac{0.18}{12}$$

$$n = 6(12) = 72$$

$$P = 16000(0.85)$$

$$E = \frac{16000(0.85)\left(0.18/12\right)}{1 - (1+0.18/12)^{-72}} = \$310.19$$

Example 4: A company <mark>anticipates need</mark>ing to replace one of their assembly line <mark>machines in 3 years at</mark> a cost of $25,000. An account at their credit union pays 4.5% per year compounded monthly. How much would they need to deposit in this account each month to reach their goal in 3 years?

Amortization
$$E = \frac{Pi}{1-(1+i)^{-n}}$$

$$i = \frac{0.045}{12}$$

$$n = 3(12) = 36$$

Sinking Fund
$$E = \frac{iF}{(1+i)^n - 1}$$

$$E = \frac{\left(\frac{0.045}{12}\right) 25000}{\left(1 + \frac{0.045}{12}\right)^{36} - 1} = \$649.92$$

Example 5: In order to expand their growing business, a <mark>company purchased a piece of property at a cash price of</mark> $450,000. They made a $25,000 down payment, and financed the rest through a lender. If the lender will charge 3.25% per year compounded <mark>semiannually for</mark> 7 years, find the company's semiannual payment.

Amortization
$$E = \frac{Pi}{1-(1+i)^{-n}}$$

$$i = \frac{0.0325}{2}$$

$$n = 2(7) = 14$$

Sinking Fund
$$E = \frac{iF}{(1+i)^n - 1}$$

$$P = 450000 - 25000 = 425000$$

$$E = \frac{425000(0.0325/2)}{1 - (1 + 0.0325/2)^{-14}} = \$34,186.03$$

Try this one: You bought an RV at a cost of $125,000. You made a down payment of $3,000. How much are your monthly payments to the creditor over 20 years at 2.25% per year compounded monthly?