Section 5.1
Sets

A collection of objects is called a **set.**

An object of a set is called an **element.** $\{1, 2, 3, 5, 6\}$

Notation:

$\in$ = "element of" belongs to

$\notin$ = "not an element of" does not belong to

$\{-1, -5, 2, 6, 1\}$
$\{2, 1, -5, 6, -1\}$ same

such that

The set C = $\{x \mid x^2 = 9\}$ is in **set builder notation**. The set C can also be written as follows: C = {-3, 3}.

$N = \{x \mid x$ is a positive even no. less than $10\}$

$N = \{2, 4, 6, 8\}$

Let *A* and *B* be two sets. If every element of *A* is also in *B*, *A* is said to be a **subset** of *B*.

Notation: $A = \{1, 2\}$ $B = \{1, 2, 3\}$

$\subseteq$ = "subset of" $A \subseteq B$

$\not\subseteq$ = "not a subset of" $B \not\subseteq A$ as B has '3'

Example 1: Let C = {1,2,3,4,5,6}, D = {2,4,6}, E = {2,1,6,4,3,5}, and G = {1, 4, 6}. State whether each of the following statements are true or false.

I. $D \subseteq C$ True

II. $E \not\subseteq C$ False $C = E$

III. $D \subseteq G$ False $2 \notin G$ and $2 \in D$

The set *A* is a **proper subset** of a set *B* (Notation: $A \subset B$) if the following two conditions hold.

1. $A \subseteq B$ $A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 4, 5\}$

$A \subset B$

2. There exists at least one element in *B* that is not in *A*.

$I \subseteq J$ $J \subseteq I$

Example 2: Let G = {5,6,7,8,9,10}, H = {5,8,10}, I = {8, 5}, and J = {5,8}. State whether each of the following statements are true or false.

I. $H \subset G$ True

II. $H \subset J$ False $H \not\subseteq J$

III. $J \subset H$ True

IV. $I \not\subset J$ True $I \subseteq J$ $I = J$

$H \subseteq G$ True

$H \not\subset G$ False

A set that contains no elements is called the **empty set.** {0} → this is not an empty set

Note: We write Ø to denote the empty set. The ~~symbol~~ Ø is a subset of every set.

{Ø} ×

Example 3: Let E = {*x*, *y*, *z*}. List all subsets of the set E.

Ø, {x}, {y}, {z}, {x,y}, {y,z}, {x,z}, {x,y,z}

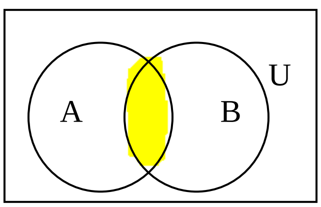
Which of the subsets of E are proper subsets?

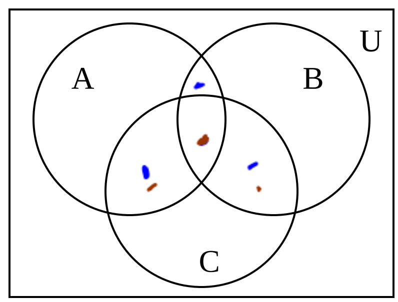
Ø, {x}, {y}, {z}, {x,y}, {y,z}, {x,z}

The **Universal set** is the set of interest in a particular discussion.

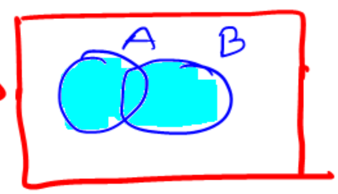
A **Venn diagram** is a visual representation of sets.

Some look like:





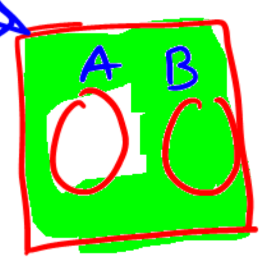
Let *A* and *B* be two sets. **Set Union**: $(A \cup B) = \{x \mid x \in A \text{ or } x \in B \text{ or both}\}$.



Let *A* and *B* be two sets. **Set Intersection:** $(A \cap B) = \{x \mid x \in A \text{ and } x \in B\}$.

If $A \cap B = \varnothing$, then we say that *A* and *B* are **disjoint**.

Let U be a universal set and $A \subseteq U$. **Complement of a Set A:** $A^c = \{x \mid x \in U, x \notin A\}$.



Some Set Operation Rules

Let U be a universal set and A and B be subset of U.

$\varnothing^c = U$	$(A^c)^c = A$	$A \cup B = B \cup A$
$U^c = \varnothing$	$(A \cup B)^c = A^c \cap B^c$	$A \cap B = B \cap A$
	$(A \cap B)^c = A^c \cup B^c$	

De Morgans Law

Example 4: Let U={1,2,3,4,5}, A = {1,2,4,5}, B = {4,5}, and C = {2,3,4}.

Find the given sets.

a. $(A \cup B)$ $= \{1,2,4,5\} \cup \{4,5\}$

$= \{1,2,4,5\}$

b. $(A \cap C)$ $= \{1,2,4,5\} \cap \{2,3,4\}$

$= \{2,4\}$

c. B^c $= \{1,2,3\}$

d. $(A \cup C^c)$ $= \{1,2,4,5\} \cup \{1,5\}$

$= \{1,2,4,5\}$

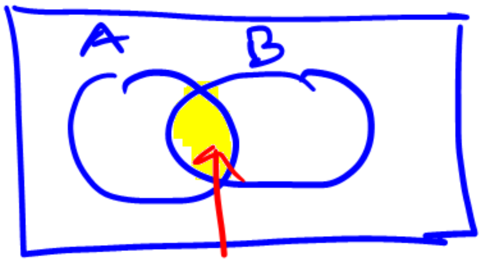
e. $(C \cap B^c)^c$ $= \{\{2,3,4\} \cap \{1,2,3\}\}^c$

$= \{2,3\}^c = \{1,4,5\}$

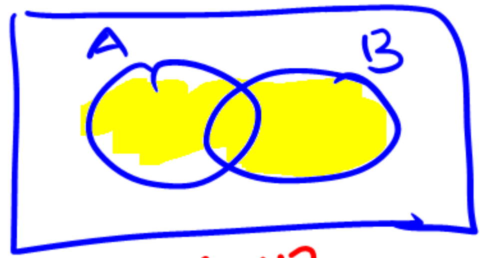
f. $(C^c \cup (B \cap A))$ $= \{1,5\} \cup (\{4,5\} \cap \{1,2,4,5\})$

$= \{1,5\} \cup \{4,5\}$

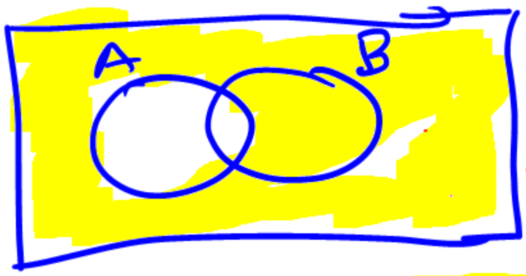
$= \{1,4,5\}$



$A \cap B$

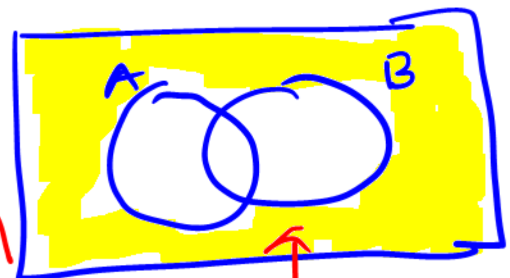


$A \cup B$



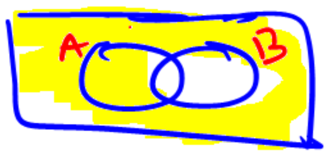
A^c

$\cap$

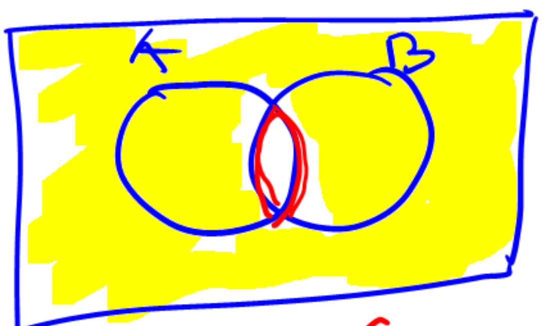


$(A \cup B)^c = A^c \cap B^c$

$\cup$

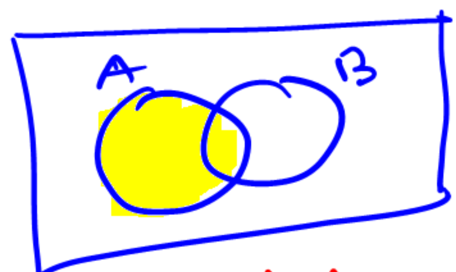


B^c

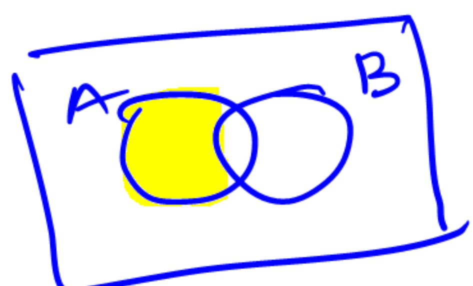


$(A \cap B)^c$

$= A^c \cup B^c$



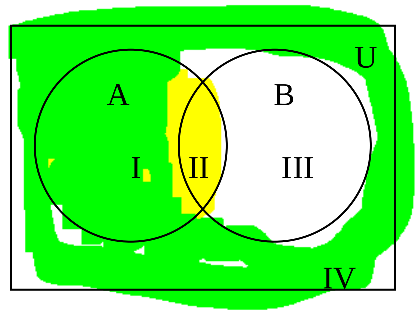
set A



only A

Example 5: Use shading to state the region(s) that represent(s) the given set. (Assume the given sets are not disjoint. This is obvious from the Venn diagrams.)

a. $(A \cap B^c)$

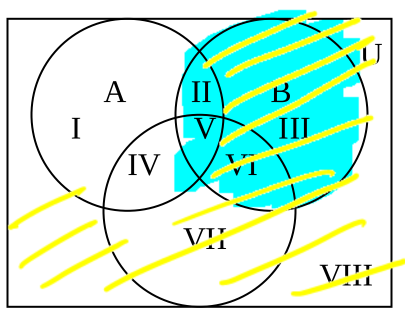


A $\quad$ B^c

$= \{I, II\} \cap \{I, IV\}$

$= \{I\}$

b. $(A^c \cup B)$



$A^c \cap B$

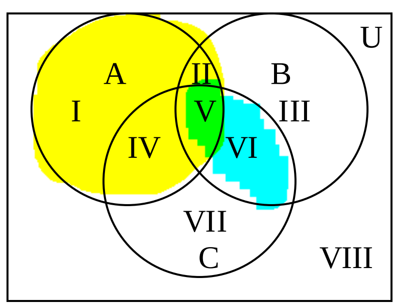
$= \{III, VI\}$

A^c

$= \{III, VI, VII, VIII\} \cup \{II, V, VI, III\}$ B

$= \{III, VI, VII, VIII, II, V\}$

c. $(A \cup (B \cap C))$

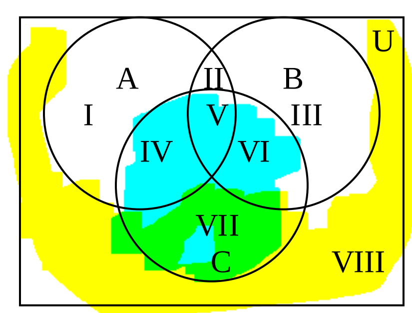


A

$= \{I, II, V, IV\} \cup \{V, VI\}$

$= \{I, II, V, IV, VI\}$

d. $((A \cup B)^c \cap C)$



$(A \cup B)^c$ $\quad$ C

$= \{VII, VIII\} \cap \{IV, V, VI, VII\}$

$= \{VII\}$

e. $((B \cap C)^c \cap A^c)$

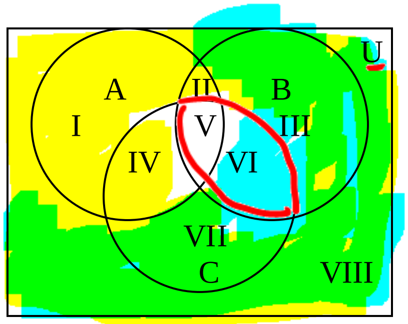
$(B \cap C)^c$

$\{I, II, III, IV, VII, VIII\}$

A^c

$\cap \{III, VI, VII, VIII\}$

$= \{III, VII, VIII\}$

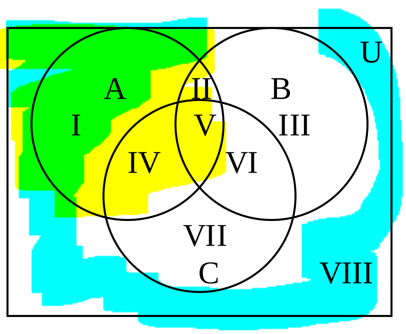


f. $((C^c \cap B^c) \cup A)$

$= (C \cup B)^c \cup A$

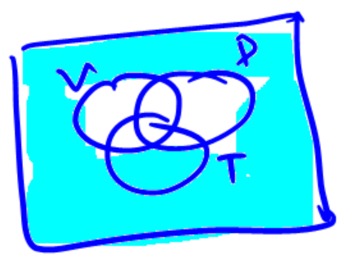
$= \{I, VIII\} \cup \{I, IV, V, II\}$

$= I, II, IV, V, VIII$



Example 6: Let U denote the set of all employees at a certain company. Let
V={$x \in$ U| x likes to read Vogue magazine},
P={$x \in$ U| x likes to read People magazine}, and
T={$x \in$ U| x likes to read Time magazine}.

Assume none of these sets are disjoint.



a. Describe the given set in words.

i. T = the set of all employees at this company that like

to read time magazine

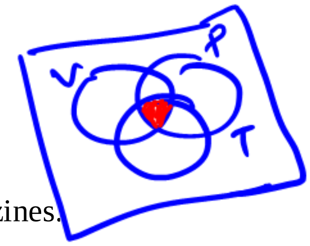
ii. $V \cap P$ = = the set of all employees at this company that like

to read both vogue & People

iii. $(P \cup V)^c$ = the set of all employees at this company that

do not like to read Vogue or People

likes to read time



b. Describe the given statement in set notation.

i. The set of all employees at this company that like to read all three magazines.

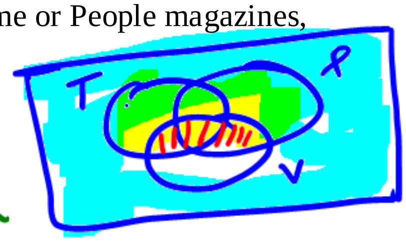
$V \cap P \cap T$

ii. The set of all employees at this company that like to read Time or People magazines.

$T \cup P$

iii. The set of all employees at this company that like to read Time or People magazines, but not Vogue.

$(T \cup P) \cap V^c$



$T \cup (P \cap V^c)$ X

iv. The set of all employees at this company that like to read only Time magazine.

$T \cap (P \cup V)^c$