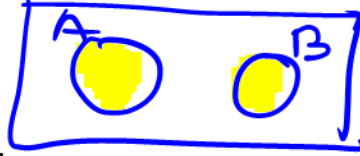


Section 5.2 Number of Elements in a Set

Let A be a set, then $n(A)$ is the **number of elements in the set A** .

Given two sets A and B .

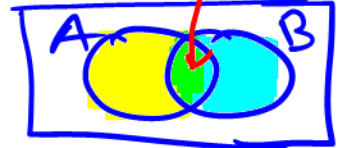
$$n(A \cap B) = 0$$



Counted twice

1. If A and B are disjoint then $n(A \cup B) = n(A) + n(B)$.

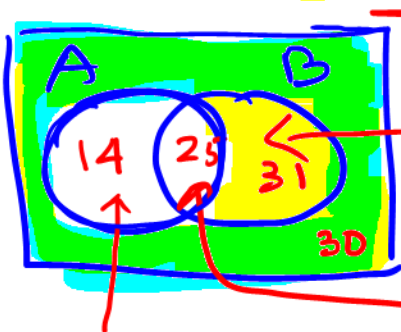
2. If A and B are not disjoint then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.



Example 1: Let A and B be subsets of a universal set U . Given that $n(B) = 9$, $n(A \cap B) = 5$, and $n(A \cup B) = 20$, find $n(A)$.

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ 20 &= n(A) + 9 - 5 \\ n(A) &= 16 \end{aligned}$$

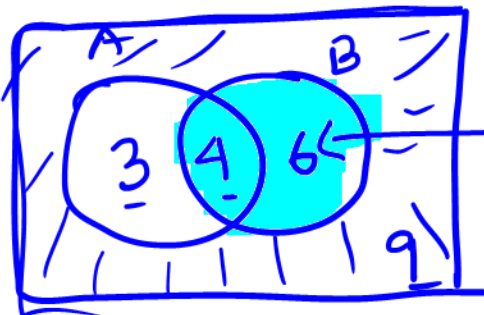
Example 2: Let A and B be subsets of a universal set U . Given that $n(U) = 100$, $n(A^c) = 61$, $n(B) = 56$, and $n(A \cup B)^c = 30$. Find $n(A^c \cap B)$ and $n(A^c \cup B)$.



$$\begin{aligned} n(A^c) - n(A \cup B)^c &= 61 - 30 = 31 \\ n(B) - n(\text{only } B) &= 56 - 31 = 25 \end{aligned} \quad \left| \begin{aligned} n(A^c \cap B) &= 31 \\ n(A^c \cup B) &= 31 + 30 + 25 = 86 \end{aligned} \right.$$

Example 3: Let A and B be subsets of a universal set U . Given that $n(U) = 22$, $n(A^c \cup B)^c = 3$, $n(A \cap B) = 4$, and $n(A \cup B)^c = 9$, find $n(B)$.

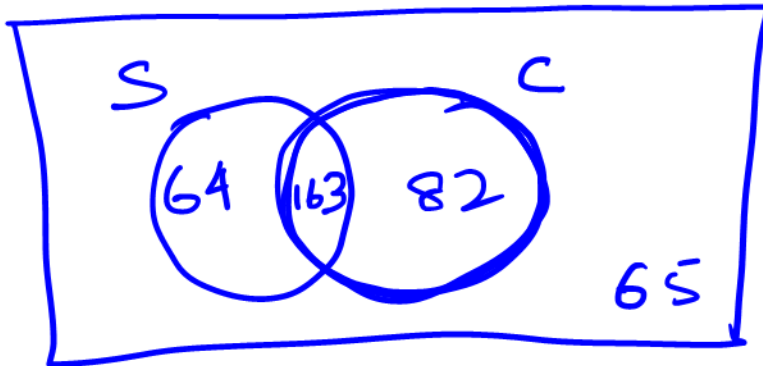
$$(A^c \cup B)^c = (A^c)^c \cap B^c = A \cap B^c$$



$$22 - (3 + 4 + 9) = 6$$

$$n(B) = 4 + 6 = 10$$

Example 4: In a survey of 374 coffee drinkers it was found that 64 take only sugar, 82 take only cream, and 65 don't take sugar nor cream with their coffee. How many take:



$$374 - (64 + 82 + 65) = 163$$

a. sugar and cream with their coffee?

$$163$$

b. cream?

$$163 + 82 = 245$$

c. neither sugar nor cream?

$$65$$

d. exactly one of these two additions? $(S \cup C) - (S \cap C)$

$$64 + 82 = 146$$

e. at least one of these two additions?

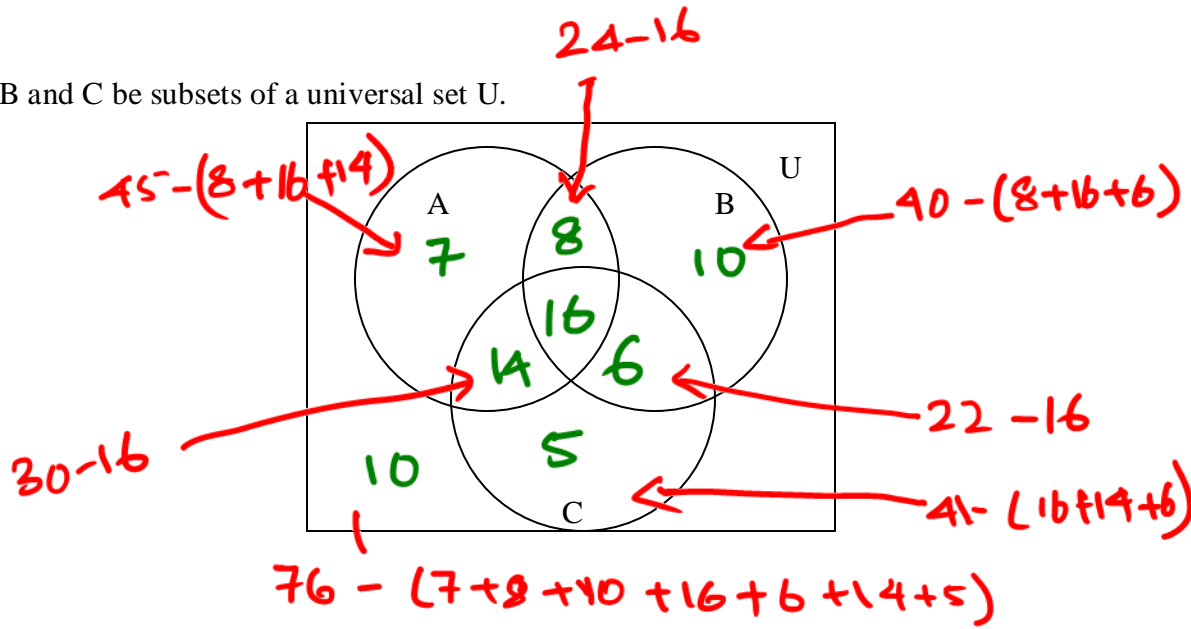
$$S \cup C$$

$$64 + 163 + 82 = 309$$

$$\text{OR } 374 - 65 = 309$$

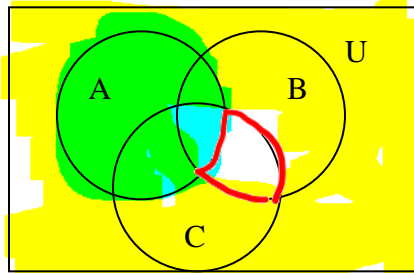
Example 5: Let A, B and C be subsets of a universal set U.

- $n(U) = 76$
- $n(A) = 45$
- $n(B) = 40$
- $n(C) = 41$
- $n(A \cap B) = 24$
- $n(B \cap C) = 22$
- $n(A \cap C) = 30$
- $n(A \cap B \cap C) = 16$.



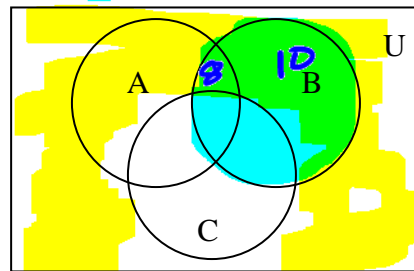
Find each of the following:

a. $n[(B \cap C)^c \cup A] =$

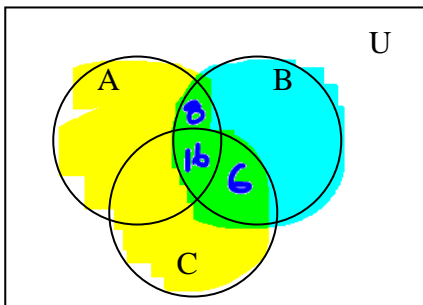


$76 - 6 = 70$

b. $n[B \cap C^c] = 8 + 10 = 18$



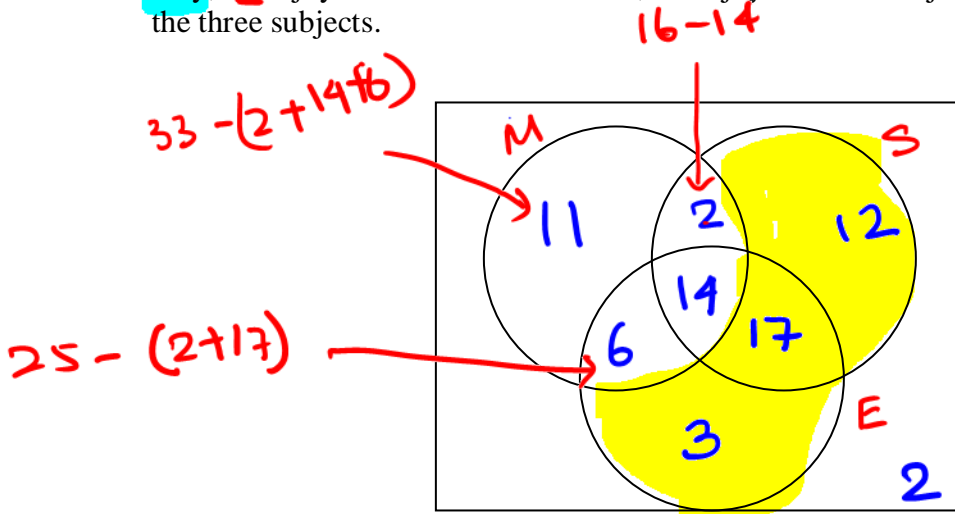
c. $n[(A \cup C) \cap B] = 8 + 16 + 6 = 30$



(d) $n[(B \cap C)^c \cap A] = 7 + 8 + 14 = 29$



Example 6: Sixty-seven students were asked about which subject they enjoy most. The survey revealed that 33 enjoy Math, 45 enjoy Science, 40 enjoy English, 17 enjoy Science and English only, 16 enjoy both Math and Science, 14 enjoy all three subjects, and 25 enjoy exactly two of the three subjects.



a. How many students surveyed enjoy Math and English?

$$6 + 14 = 20$$

$$\text{only}(M \& E) = 6$$

b. How many students surveyed enjoy Science or English but not Math?

$$12 + 17 + 3 = 32$$

$$(S \cup E) \cap M^c$$

c. How many students surveyed enjoy at most one of the three subjects mentioned?

can like maximum 1 $\rightarrow 11 + 12 + 3$
 cannot like any $\rightarrow 2$

$$11 + 12 + 3 + 2 = 28$$