Section 5.4
Permutations and Combinations

$\frac{5!}{2!} = \frac{\cancel{1}\cdot\cancel{2}\cdot 3\cdot 4\cdot 5}{\cancel{1}\cdot\cancel{2}} = 60$

Definition: n-Factorial

For any natural number n, $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$.

0! = 1 $5! = 1.2.3.4.5 = 120$ $20! = 1.2\ldots 19.20$

A **combination** of a set is arranging the elements of the set without regard to order.

Example: The marinade for my steak contains soy sauce, Worchester sauce and a secret seasoning.

${}^nC_r$

Formula: $C(n, r) = \dfrac{n!}{r!(n-r)!}$, $r \le n$, where n is the number of distinct objects and r is the number of distinct objects taken r at a time.

$C(5,2) = \frac{5!}{2!\,3!} = \frac{1.\cancel{2}.\cancel{3}.4.5}{1.\cancel{2}.1.\cancel{2}.\cancel{3}} = 10$

$5C2$

A **permutation** of a set is arranging the elements of the set with regard to order.

Example: My previous pin number was 2468, now it's 8642.

${}^nP_r$

Formula: $P(n, r) = \dfrac{n!}{(n-r)!}$, $r \le n$, where n is the number of distinct objects and r is the number of distinct objects taken r at a time.

$P(5,2) = \frac{5!}{3!} = \frac{1.\cancel{2}.\cancel{3}.4.5}{1.\cancel{2}.\cancel{3}} = 20$

$5P2$

The deck of 52 playing cards is a good set to use with some of these problems, so let's make some notes:

52 total cards, no jokers--26 red and 26 black
Suits are Hearts, Diamonds, Clubs, and Spades.
Each suit has 13 cards, one of each 2 – 10, Jack, Queen, King, Ace
Face cards are J, Q, K only = 12 face cards

2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥	8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥	A ♥
2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦	8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦	A ♦
2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣	8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	A ♣
2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠	8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	A ♠

Number: 2-10 9
Face Cards: K, Q, J 3
Ace 1

Example 1: In how many ways can 7 cards be drawn from a well-shuffled deck of 52 playing cards?

Combination or Permutation

$C(52,7) = 133784560$

Example 2: An organization has 30 members. In how many ways can the positions of president, vice-president, secretary, treasurer, and historian be filled if not one person can fill more than one position?
Combination or Permutation

$P(30,5) = 17100720$

Example 3: In how many ways can 10 people be assigned to 5 seats?
Combination or Permuation

$P(10,5) = \frac{10!}{(10-5)!} = \frac{10!}{5!} = 30240$

Example 4: An organization needs to make up a social committee. If the organization has 25 members, in how many ways can a 10 person committee be made?
Combination or Permuation

$C(25,10) = \frac{25!}{10!\,15!}$

$C(25,10) = 3268760$

Example 5: Seven people arrive at a ticket counter at the same time to buy concert tickets. In how many ways can they line up to purchase their tickets?
Combination or Permuation

$P(7,7) = 5040$

7 6 5 4 3 2 1

7!

Formula: Permutations of n objects, not all distinct

Given a set of n objects in which n_1 objects are alike and of one kind, n_2 objects are alike and of another kind,…, and, finally, n_r objects are alike and of yet another kind so that

$$n_1 + n_2 + ... + n_r = n$$

$P(6,6) = 6!$

then the number of permutations of these n objects taken n at a time is given by

FUNNY

$\frac{5!}{2!}$

$$\frac{n!}{n_1!n_2!\cdots n_r!}$$

FLOWER

$6! = 720$

Example: All arrangements that can be made using all of the letters in the word COMMITTEE.

$\frac{9!}{2!\,2!\,2!} = \frac{9!}{8} = 45360$

Example 6: REENNER, a small software company would like to make letter codes using all of the letters in the word REENNER. How many codes can be made from all the letters in this word?
Combination or Permuation

$\frac{7!}{2!\,3!\,2!} = 210$

H H H H H
T T T T H
T H T H T
T T H H H
H H H T T

Example 7: A coin is tossed 5 times.

a. How many outcomes are possible?

$\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}\ \underline{2} = 2^5 = 32$

b. In how many outcomes do exactly 3 heads occur? $C(5,3) = 10$

{ ($H_1H_2H_3TT$), ($H_1H_2T H_4T$), ($H_1H_2TT H_5$), ($H_1TH_3T H_5$), ($H_1T TH_4H_5$), ($H_1T H_3H_4T$), ($TH_2H_3H_4T$), ($T H_2H_3T H_4$), ($TH_2TH_4H_5$), ($T TH_3H_4H_5$) }

c. In how many outcomes do exactly 2 tails occur?

$C(5,2) = 10$

$C(n, n-1) = n$

ex $C(5,4) = 5$

$C(n,n) = 1$

ex $C(5,5) = 1$

$C(n,0) = 1$

ex $C(5,0) = 1$

$C(n,1) = n$

ex $C(5,1) = 5$

Example 8: A coin is tossed 18 times. {H, T}

a. How many outcomes are possible?

2^{18}

b. In how many outcomes do exactly 7 tails occur?

$C(18,7)$

c. In how many outcomes do at most 2 tails occur? 0T, 1T, 2T

$C(18,0) + C(18,1) + C(18,2) = 172$

$= 1 + 18 + 153$

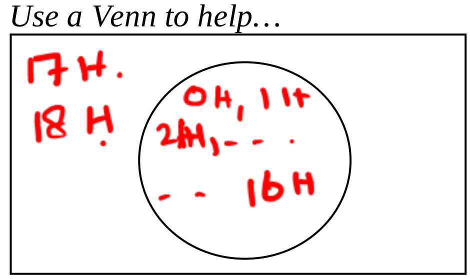
d. In how many outcomes do at least 17 tails occur? 17T, 18T

$C(18,17) + C(18,18) = 19$

$= 18 + 1$

e. In how many outcomes do at most 16 heads occur?

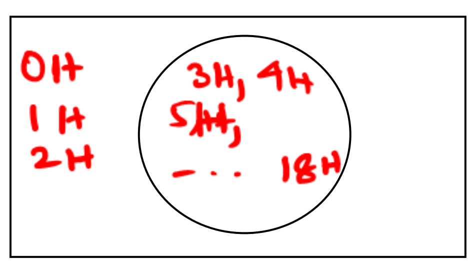
Use a Venn to help…



$2^{18} - (C(18,17) + C(18,18))$

$= 2^{18} - 19 = 262125$

f. In how many outcomes do at least 3 heads occur?



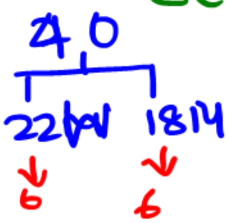
$2^{18} - (C(18,0) + C(18,1) + C(18,2))$

$= 2^{18} - 172 = 261972$

Example 9: A judge has a jury pool of 40 people that contains 22 women and 18 men. She needs a jury of 12 people.

a. How many juries can be made?

$C(40,12) = 5586853480$



b. How many juries contain 6 women and 6 men?

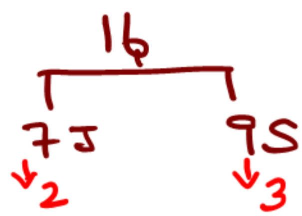
$C(22,6) \cdot C(18,6)$

$= 1385115732$

Example 10: A club of 16 students, 7 juniors and 9 seniors, is forming a 5 member subcommittee.

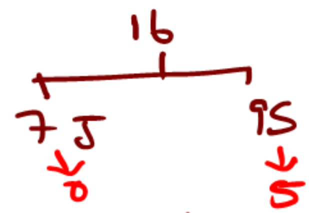
a. How many subcommittees can be made?

$C(16,5)$



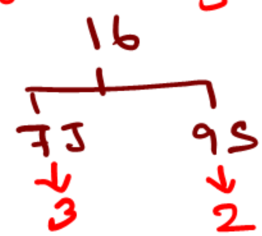
b. How many subcommittees contain 2 juniors and 3 seniors?

$C(7,2) \cdot C(9,3)$



c. How many subcommittees contain all seniors?

$C(9,5)\, C(7,0) = C(9,5)$



d. How many subcommittees contain 3 juniors?

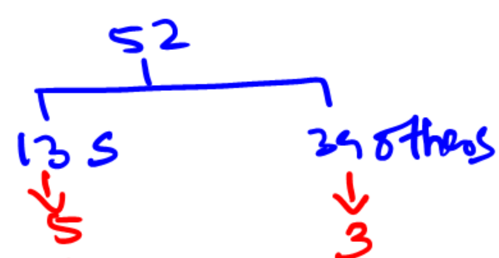
$C(7,3) \cdot C(9,2) = 1260$

Example 11: In how many ways can 5 spades be chosen if 8 cards are chosen from a well-shuffled deck of 52 playing cards?

2 ♡	3 ♡	4 ♡	5 ♡	6 ♡	7 ♡	8 ♡	9 ♡	10 ♡	J ♡	Q ♡	K ♡	A ♡
2 ♢	3 ♢	4 ♢	5 ♢	6 ♢	7 ♢	8 ♢	9 ♢	10 ♢	J ♢	Q ♢	K ♢	A ♢
2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣	8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣	A ♣
2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠	8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠	A ♠

spades = 13

Rest = 52 − 13 = 39



$C(13,5) \cdot C(39,3)$

$= 11761893$

Example 12: A customer at a fruit stand picks a sample of 7 oranges at random from a crate containing 35 oranges of which 5 are rotten.

a. How many selections can be made?

$C(35,7)$

35

30G 5R

3 4

b. How many selections contain 4 rotten?

$C(30,3)\cdot C(5,4)$

c. How many selections contain at least 4 rotten?

4R,3G , 5R 2G

$C(5,4)\cdot C(30,3) + C(5,5)\cdot C(30,2)$

d. How many selections contain at most 4 rotten? 0R, 1R, 2R, 3R, 4R

complemen 5R

$C(35,7) - [C(30,2)\cdot C(5,5)]$

(5,2) (0,7) (1,6) ... (4,3)

e. How many selections contain at least 1 rotten?

1R 6G
2R 5G
3R 4G
4R 3G
5R 2G

0R 7G

$C(35,7) - C(30,7)$

f. How many selections contain at least 3 rotten?

$C(5,3)\cdot C(30,4)$
$+ C(5,4)\, C(30,3)$
$+ C(5,5)\cdot C(30,2)$

3R 4G
4R 3G
5R 2G

Example 13: A store receives a shipment of 35 calculators including 8 that are defective. A sample of 6 calculators is chosen at random. How many selections contain at least 5 defective calculators?

5D oR 6D

$C(27,1)\cdot C(8,5) + C(27,0)\cdot C(8,6)$

35

27G 8D

1 5

0 6