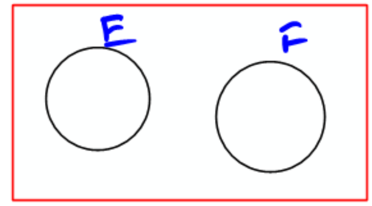
Section 6.3
Rules of Probability



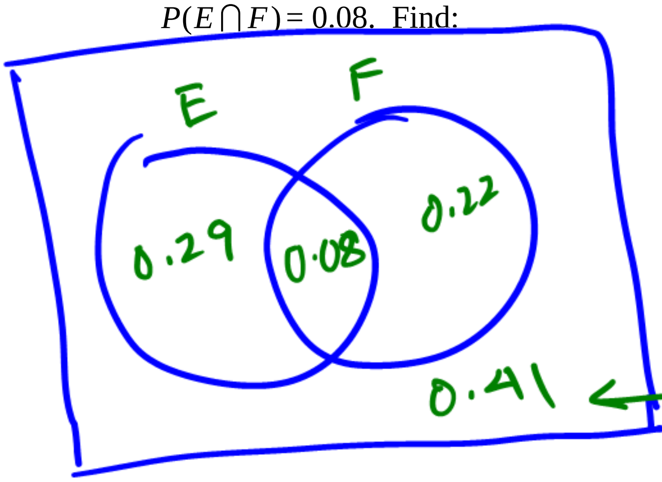
Let S be a sample space, E and F are events of the experiment then,

1. P(E$\cup$F) = P(E) + P(F) *if E and F are mutually exclusive, (E$\cap$F = Ø).*
(Note that this property can be extended to a finite number of events.)

2. P(E$\cup$F) = P(E) + P(F) – P(E$\cap$F) *if E and F are not mutually exclusive, (E$\cap$F ≠ Ø).*
(Note that this property can be extended to a finite number of events.)

3. Rule of Complements: P(E^c) = 1 – P(E) or P(E) = 1 – P(E^c)

Example 1: Let E and F be two events and suppose that P(E) = 0.37, P(F) = 0.3 and $P(E\cap F) = 0.08$. Find:



P(only E) = 0.37 – 0.08
= 0.29

P(only F) = 0.3 – 0.08
= 0.22

0.41 ← 1 – (0.29 + 0.08 + 0.22)

a. P(E$\cup$F) = 0.29 + 0.08 + 0.22 = 0.59

OR 1 – 0.41 = 0.59

b. $P(E^c \cap F)$

0.22

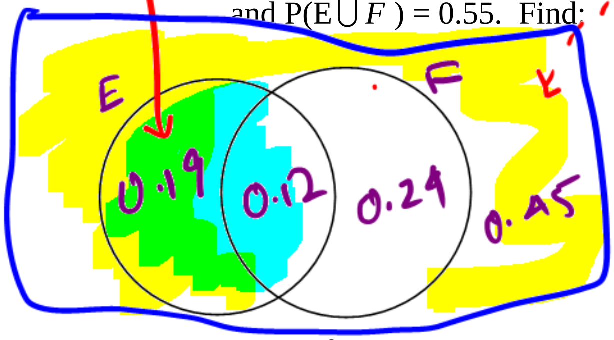
c. P(F^c) = 0.29 + 0.41 OR P(F^c) = 1 – P(F)
= 0.7 = 1 – 0.3 = 0.7

d. $P(E\cap F)^c$ = 0.29 + 0.22 + 0.41 OR P($E\cap F)^c$ = 1 – P(E$\cap$F)
= 0.92 = 1 – 0.08
= 0.92

$P(\text{only } E) = 0.31 - 0.12$
$= 0.19$

$1 - (0.19 + 0.12 + 0.24)$

Example 2: Let E and F be events of a sample space S. Let $P(E^c) = 0.69$, $P(F) = 0.36$ and $P(E \cup F) = 0.55$. Find:



$P(E) = 1 - P(E^c) = 1 - 0.69 = 0.31$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$0.55 = 0.31 + 0.36 - P(E \cap F)$
$P(E \cap F) = 0.12$

a. $P(E \cap F^c)$

$= 0.19$

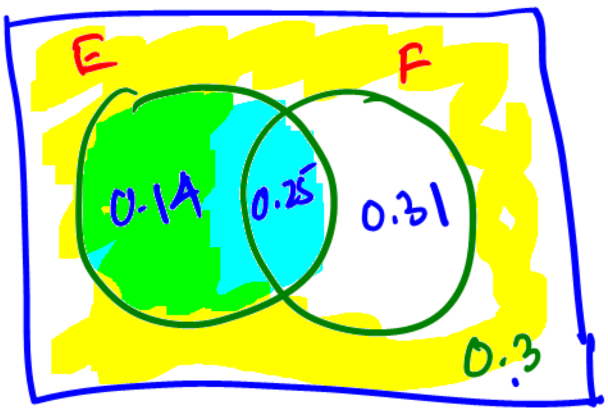
b. $P(E^c \cup F)$

$= 0.24 + 0.45 + 0.12$
$= 0.81$

$P(\text{only } F) = P(E^c) - P(E \cup F)^c$

$(E^c \cup F)^c = E \cap F^c$

Try this one: Let E and F be events of a sample space S. Let $P(E^c) = 0.61$, $P(F^c) = 0.44$, and $P(E \cup F)^c = 0.3$. a. Find $P(E^c \cup F^c)^c$. b. Find $P(E^c \cup F)^c$



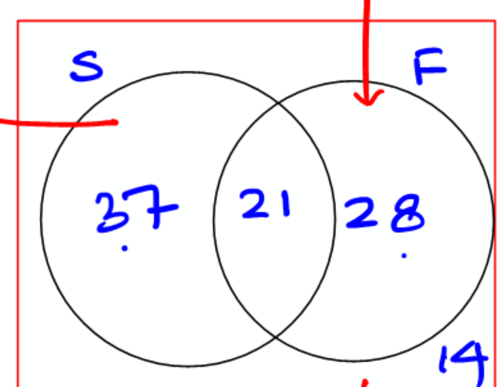
$P(E) = 1 - P(E^c) = 1 - 0.61 = 0.39$
$P(F) = 1 - P(F^c) = 1 - 0.44 = 0.56$
$P(E \cup F) = 1 - P(E \cup F)^c = 1 - 0.3 = 0.7$
$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$0.7 = 0.39 + 0.56 - P(E \cap F)$
$P(E \cap F) = 0.25$
$P(E^c \cup F^c)^c = P(E \cap F) = 0.25$

$P(E^c \cup F)^c$
$= P(E \cap F^c)$
$= 0.14$

Example 3: One-hundred students were asked if they speak Spanish or French. It was found that 58 speak Spanish, 49 speak French, and 21 speak both languages. If a student is chosen at random, what is the probability that he or she can speak:

$49 - 21$

$58 - 21$



$100 - (37 + 21 + 28)$

a. French only?

$28/100 = 0.28$

1 or 2

b. at least one of the two languages mentioned?

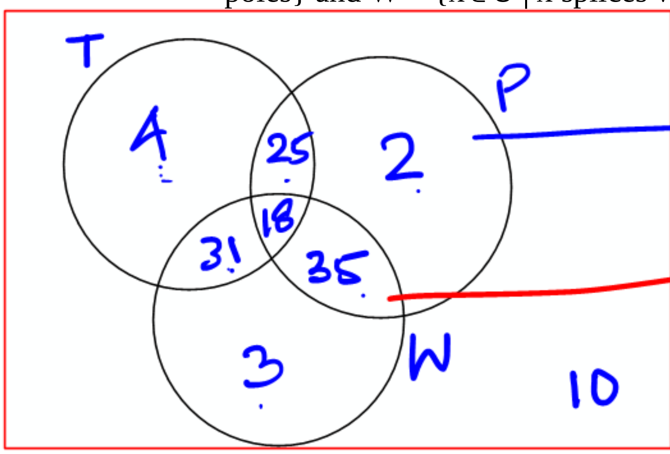
$(37 + 28 + 21)/100 = 86/100 = 0.86$

c. either Spanish or French, but not both?

$(37 + 28)/100 = 65/100 = 0.65$

Example 4: You are a chief for an electric utility company. The employees in your section cut down trees, climb poles, and splice wire. You report that of the 128 employees in your department 10 cannot do any of the three (management trainees), 25 can cut trees and climb poles only, 31 can cut trees and splice wire, but not climb poles, 18 can do all three, 4 can cut trees only, 3 can splice wire, but cannot cut trees or climb poles, and 9 can do exactly one of the three.

Let U = 128 employees at this company, T = {x ∈ U | x cuts trees}, P = {x ∈ U | x climbs poles} and W = {x ∈ U | x splices wire}.



$9-(4+3)$

$128-[4+25+18+31+3+10+2]$

An employee is selected at random. What is the probability that the employee:

a. can do at exactly one of the three jobs mentioned here?

$$[4+2+3]/128 = \frac{9}{128} \approx 0.07$$

b. can do at most one of the three jobs mentioned here?

0 or 1

$$[10+9]/128 = \frac{19}{128} \approx 0.15$$

c. can climb poles and splice wire?

$$[18+35]/128 = \frac{53}{128} \approx 0.41$$

d. at least two of the three jobs mentioned here?

$$[31+25+35+18]/128 = \frac{109}{128} \approx 0.85$$