**Section 6.4**
**Use of Counting Techniques in Probability**

Let S be a uniform sample space and let E be any event. Then, $P(E) = \frac{n(E)}{n(S)}$.

Example 1: Consider the experiment of tossing a fair coin 10 times. $2^{10} = 1024$

a. Find the probability that the coin lands heads exactly 7 times.

# times lands 7 heads exactly $= C(10,7) = 120$

$P(7H) = \frac{120}{2^{10}} = \frac{120}{1024} \approx 0.12$

b. Find the probability that the coin lands heads at most 2 times.

0H, 1H, 2H

$C(10,0) + C(10,1) + C(10,2) = 56$

$P(\text{at most } 2H) = \frac{56}{1024} \approx 0.05$

c. Find the probability that the coin lands tails at least 9 times.

9T or 10T

$C(10,9) + C(10,10) = 11$

$P(\text{at least } 9T) = \frac{11}{1024} \approx 0.01$

d. Find the probability that the coin lands tails at least once.

1T or 2T or 3T - - - 10T

0T can occur $= C(10,0) = 1$

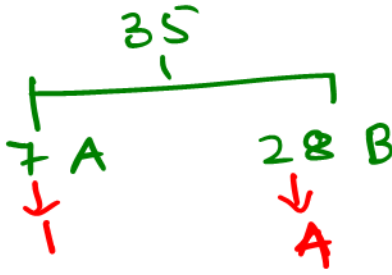
At least one tail will occur $= 1024 - 1 = 1023$

$P(\text{at least } 1T) = \frac{1023}{1024} \approx 0.999$

Example 2: In a survey of 35 consumers at a local supermarket, 7 indicated that they buy brand A of a certain product and the rest indicated that they buy brand B of the same product. You choose 5 customers surveyed at random.

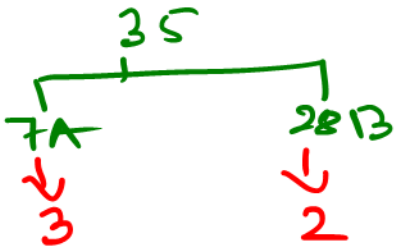
a. What is the probability that 4 buy brand B and 1 buys brand A?

$$\frac{C(7,1)\cdot C(28,4)}{C(35,5)} \approx 0.44$$




b. What is the probability that 3 buy brand A?

$$\frac{C(7,3)\cdot C(28,2)}{C(35,5)} \approx 0.04$$

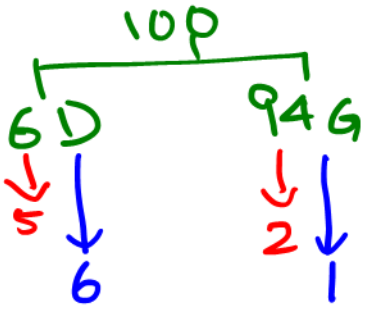



Example 3: A department store is shipped 100 remote controlled cars of which 6 are defective. A customer selects 7 cars at random.

a. What is the probability that at least 5 will be defective?

5 or 6

$$\frac{C(6,5)\cdot C(94,2) + C(6,6)\,C(94,1)}{C(100,7)} \approx 0.0000016$$




b. What is the probability that at most 4 will be defective?

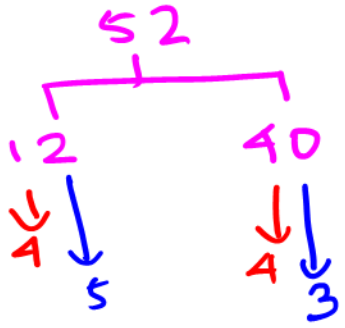
0 or 1 or 2 or 3 or 4 OR 5 or 6 D

$$\frac{C(100,7) - [C(6,5)\cdot C(94,2) + C(6,6)\,C(94,1)]}{C(100,7)} \approx 0.999998$$

Example 4: Eight cards are selected at random from a well-shuffled deck of 52 playing cards.

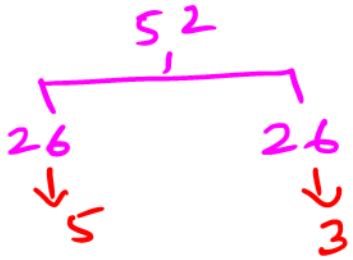
a. What is the probability that either 4 face cards or 5 face cards are chosen?

$$\frac{C(12,4)\cdot C(40,4) + C(12,5)\cdot C(40,3)}{C(52,8)} \approx 0.0705$$



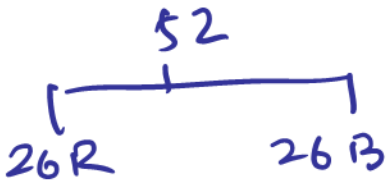

b. What is the probability that 5 cards are red?

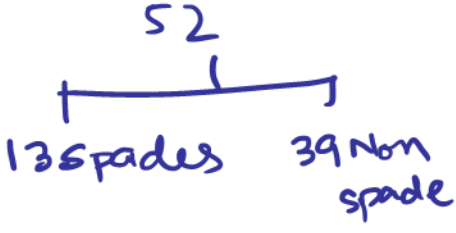
$$\frac{C(26,5)\cdot C(26,3)}{C(52,8)} \approx 0.227$$




c. What is the probability that 3 cards are red or 4 cards are spade?

$$\frac{C(26,3)\,C(26,5) + C(13,4)\,C(39,4)}{C(52,8)}$$



(*) A bag contains 6 black marbles and 8 whilte marbles. 2 marbles are chosen one at a time without replacement. What is the probability that the second one is white?

$$(B, W) \qquad (W, W)$$

$$\frac{6}{14}\cdot\frac{8}{13} + \frac{8}{14}\cdot\frac{7}{13} \approx 0.571$$

$$= \frac{48}{14\cdot 3} + \frac{56}{14\cdot 13} = \frac{1}{14\cdot 13}(48+56)$$

(*) There are two bags A and B . A has 6 black and 8 white marbles and B has 5 balck and 9 white marbles. One bag is chosen at random and a marble is drawn. What is the probablity the marble is black?

$$A \qquad B$$

$$\frac{1}{2}\cdot\frac{6}{14} + \frac{1}{2}\cdot\frac{5}{14} = \frac{11}{28}$$

$$\frac{1}{C(2,1)}\frac{C(6,1)}{C(14,1)} + \frac{1}{C(2,1)}\frac{C(5,1)}{C(14,1)}$$