Section 6.6
Bayes' Theorem

In Section 6.5 we discussed the conditional probability of the occurrence of an event, given the occurrence of an earlier event. Now we are going to reverse the problem and try to find the probability of an earlier event conditioned on the occurrence of a later event. We can use a principle called Bayes' Theorem to solve problems of this type. This theorem is an application of conditional probability. We'll use the conditional probability formula and the product rule to solve these problems.

Example 1: A company produces 1,000 refrigerators a week at three plants. Plant A produces 350 refrigerators a week, plant B produces 250 refrigerators a week, and plant C produces 400 refrigerators a week. Production records indicate that 5% of the refrigerators produced at plant A will be defective, 3% of those produced at plant B will be defective, and 7% of those produced at plant C will be defective. All the refrigerators are shipped to a central warehouse.

$\frac{350}{1000}$ — A: D 0.05, G 0.95

$\frac{250}{1000}$ — B: D 0.03, G 0.97

$\frac{400}{1000}$ — C: D 0.07, G 0.93

a. What is the probability that a refrigerator chosen at random from the warehouse will be defective?

$$P(D) = (0.35)(0.05) + (0.25)(0.03) + (0.4)(0.07) = 0.053$$

b. If a refrigerator at the warehouse is found to be defective, what is the probability that it was produced at plant C?

$$P(C \mid D) = \frac{P(C \cap D)}{P(D)} = \frac{(0.4)(0.07)}{\underbrace{(0.053)}_{\text{Part (a)}}} = 0.5283$$

c. What is the probability that refrigerator is defective, given that it was produced at plant C?

$$P(D \mid C) = 0.07 \quad \text{OR} \quad P(D \mid C) = \frac{P(D \cap C)}{P(C)} = \frac{(0.4)(0.07)}{0.4} = 0.07$$

Example 2: The sales department of the Halgreens company released the accompanying data concerning the sales of a certain pain reliever manufactured by the company:

Pain Reliever	Percentage of Drug Sold	Percentage of Group Sold in Extra-Strength Dosage
Capsule Form	0.57	0.38
Tablet Form	0.43	0.31

Given that a customer purchased the extra-strength dosage of this drug, what is the probability that it was in capsule form?

$$P(\text{cap form} \mid \text{\% in extra dosage}) = \frac{P(\text{cap form} \cap \text{\% in extra dos})}{P(\text{\% in extra dos})}$$

$$= \frac{0.38}{0.38+0.31} = 0.55$$

Example 3: A new, inexpensive skin test is devised for detecting tuberculosis (TB). To evaluate the test before it is put into use, a medical researcher randomly selects 1,000 people, 8% of whom were known to have TB. Each of the 1,000 subjects is given the new skin tests and the following results are recorded: The test indicates TB in 96% of those who have it and in 2% of those who do not. A person is selected at random. What is the probability that:

actual

Test results

0.08 — TB — TB 0.96
NTB 0.04

0.92 — NTB — TB 0.02
NTB 0.98

a. the test indicates TB?

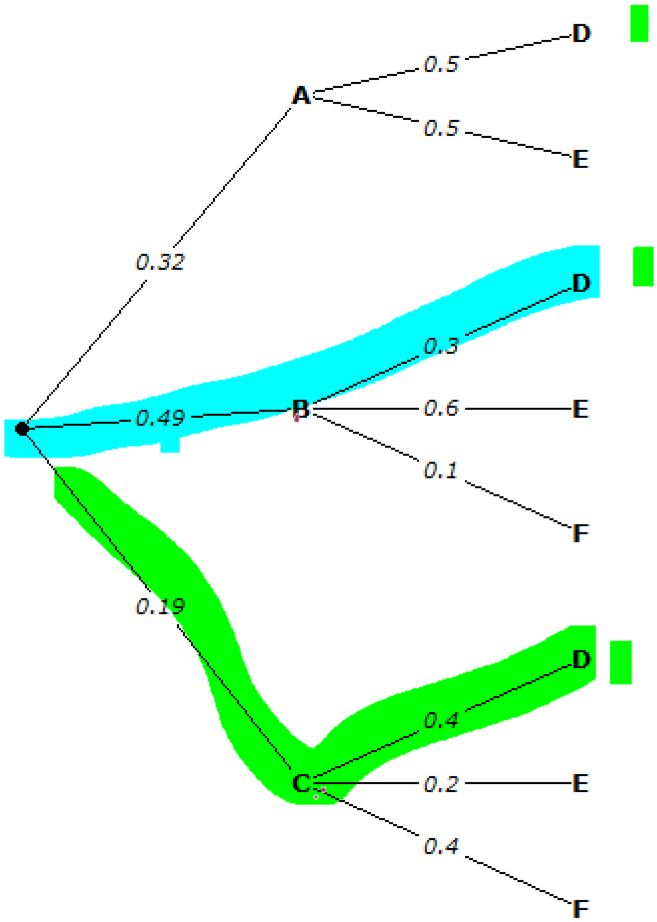
$$P(\text{test TB}) = (0.08)(0.96) + (0.92)(0.02) = 0.0952$$

b. the person does not have TB, given that the test does not indicate TB?

$$P(\text{actual NTB} \mid \text{test NTB}) = \frac{P(\text{actual NTB} \cap \text{test NTB})}{P(\text{test NTB})}$$

$$= \frac{(0.92)(0.98)}{1-0.0952} = \frac{0.9016}{0.9048} = 0.9965$$

Example 4: Given the following tree diagram, answer the given questions.



a. $P(A \cap E) = (0.32)(0.5) = 0.16$

e. $P(D \mid B) = 0.3$

b. $P(C \cap F) = (0.19)(0.4) = 0.076$

f. $P(F \mid C) = 0.4$

c. $P(D)$

$= (0.32)(0.5) + (0.49)(0.3) + (0.19)(0.4)$

$= 0.383$

g. $P(B \mid D) = \dfrac{P(B \cap D)}{P(D)} = \dfrac{(0.49)(0.3)}{0.383} = 0.3838$

d. $P(F)$

$= (0.49)(0.1) + (0.19)(0.4)$

$= 0.125$

h. $P(C \mid F) = \dfrac{P(C \cap F)}{P(F)} = \dfrac{(0.19)(0.4)}{0.125} = 0.608$