Section 7.2
Expected Value and Odds

The **average** (mean) of n numbers, $x_1, x_2, x_3, \ldots, x_n$ is $\bar{x}$.

1, 3, 6, 2, 5, 7

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

$$\bar{x} = \frac{1+3+6+2+5+7}{6} = \frac{24}{6} = 4$$

Expected Value of a Random Variable *X*

Let X denote a random variable that assumes the values $x_1, x_2, x_3, \ldots, x_n$ with associated probabilities $p_1\ p_2, \ldots, p_n$, respectively. The **expected value** of X, $E(X)$, is given by

$$E(X) = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n$$

x	$P(x)$
0.01	8/35
0.05	5/35
0.1	7/35
0.25	15/35

Example 1: A box contains 15 quarters, 7 dimes, 5 nickels, and 8 pennies. A coin is drawn at random from the box. What is the mean of the value of the draw?

$$\frac{(15)(0.25) + (7)(0.1) + (5)(0.05) + (8)(0.01)}{15 + 7 + 5 + 8} = 0.1366$$

Example 2: In a certain math class each of 4 tests are worth 12%, the quiz average is worth 12%, the popper average is worth 16%, and the final exam is worth 24%. If a student's grades are as follows:

Test 1 – 82, Test 2 – 76, Test 3 – 87, Test 4 – 90, Quiz Average – 100,
Popper Average – 100, and Final Exam – 95.

What is the students class average (expected value)?

$$E(X) = \underset{T_1}{x_1 P_1} + \underset{T_2}{x_2 P_2} + \underset{T_3}{x_3 P_3} + \underset{T_4}{x_4 P_4} + \underset{Q}{x_5 P_5} + \underset{P}{x_6 P_6} + \underset{F}{x_7 P_7}$$

$$= 82(0.12) + 76(0.12) + 87(0.12) + 90(0.12) + 100(0.12) + 100(0.16) + 95(0.24)$$

$$= 91$$

Example 3: An investor is interested in purchasing an apartment building containing six apartments. The current owner provides the following probability distribution indicating the probability that the given number of apartments will be rented during a given month.

Number of Rented Apt	0	1	2	3	4	5	6
Probability	0.02	0.08	0.10	0.12	0.15	0.25	0.28

← Random var.

a. Find the number of apartments the investor could *expect* to be rented during a given month.

$E(x) = 0(0.02) + 1(0.08) + 2(0.1) + 3(0.12) + 4(0.15) + 5(0.25) + 6(0.28)$
$= 4.17$

b. If the monthly rent for each apartment is $799, how much could the investor expect to collect in rent for the whole building during a given month?

$4(799) = 3196$

Odds in Favor of and Odds Against

If P(E) is the probability of an event E occurring, then

$P(E^c) \to$ event E not occurring

1. The odds in favor of E occurring are $\dfrac{P(E)}{P(E^c)}$, $P(E^c) \neq 0$.

2. The odds against E occurring are $\dfrac{P(E^c)}{P(E)}$, $P(E) \neq 0$.

Note: Odds are expressed as ratios of whole numbers.

Example 4: The probability of an event E occurs is 0.73. Find the odds in favor of E occurring and the odds against E occurring. $P(E) = 0.73$ $P(E^c) = 0.27$

Favor: $\dfrac{P(E)}{P(E^c)} = \dfrac{0.73}{0.27} = \dfrac{73}{27}$ Against: $\dfrac{P(E^c)}{P(E)} = \dfrac{27}{73}$

Example 5: The probability that the race horse Galloping Genny will win a race is 0.86. What are the odds Genny will not win?

$P(W) = 0.86$
$P(L) = 0.14$

$\dfrac{P(L)}{P(W)} = \dfrac{0.14}{0.86} = \dfrac{14}{86} = \dfrac{7}{43}$

7 to 43

Probability of an Event (Given the Odds)

If the odds in favor of an event E occurring are *a* to *b*, then the probability of E occurring is

$$P(E) = \frac{a}{a+b}$$

$$P(E^c) = 1 - P(E) = \frac{b}{a+b}$$

Example 6: The odds in favor of an event E occurring are 10 to 5. Find the probability that the event occurs and that the event does not occur. a b

Occurs: $P(E) = \frac{10}{10+5} = \frac{10}{15} = 0.667$

Does not occur: $P(E^c) = 1 - P(E) = 1 - 0.667 = 0.333$

OR

$P(E^c) = \frac{5}{15} = 0.333$

Example 7: The odds in favor of a certain baseball team winning the pennant are 2:7. What is the probability that that team will not win the pennant? win not win

$$P(\text{not win}) = \frac{7}{2+7} = \frac{7}{9} = 0.778$$

$$P(W) = \frac{2}{9} = 0.222$$