

Section 7.3 Variance and Standard Deviation

The **Variance** of a random variable X is the measure of degree of dispersion, or spread, of a probability distribution about its mean (i.e. how much on average each of the values of X deviates from the mean.). Note: A probability distribution with a small (large) spread about its mean will have a small (large) variance.

Variance of a Random Variable X

Suppose a random variable has the probability distribution

x	x_1	x_2	\dots	x_n
$P(X=x)$	p_1	p_2	\dots	p_n

and expected value $E(X) = \mu$. Then the **variance** of the random variable X is

$$Var(X) = p_1(x_1 - \mu)^2 + p_2(x_2 - \mu)^2 + \dots + p_n(x_n - \mu)^2$$

Note: We square each since some may be negative.

Standard Deviation measures the same thing as the variance. The standard deviation of a random variable X is

$$\sigma = \sqrt{Var(X)}$$

sigma

Note: As stated above the standard deviation measures the same thing as the variance. Since we square the $(x_i - \mu)$ s in the variance formula the units of x are squared, so to remedy this we take the square root.

Example 1: Given the following probability distribution, find its mean, variance and standard deviation.

$$E(x) = -2(0.25) + 1(0.58) + 3(0.17) = 0.59$$

x	$P(X=x)$
-2	0.25
1	0.58
3	0.17

$$Var(x) = 0.25(-2 - 0.59)^2 + 0.58(1 - 0.59)^2 + 0.17(3 - 0.59)^2 = 2.7619$$

$$\sigma = \sqrt{2.7619} = 1.6619$$

Example 2: You record the length of time in minutes it takes you to drive from your home to school in 10 consecutive days. The probability distribution follows. Let the random variable X denote the number of minutes it takes you to drive from home to school. Calculate the mean and standard deviation.

Mean:

$$E(x) = 0.2(45) + 0.3(48) + 0.5(51) = 48.9$$

x	P(X=x)
45	0.2
48	0.3
51	0.5

Standard Deviation:

$$\text{Var}(x) = 0.2(45-48.9)^2 + 0.3(48-48.9)^2 + 0.5(51-48.9)^2 = 5.49$$

$$\sigma = \sqrt{5.49} = 2.34$$

Example 3: An investor is considering two business ventures. The anticipated returns (in thousands of dollars) of each venture are described by the following probability distributions:

Venture 1		Venture 2	
Earnings	Probability	Earnings	Probability
-5	0.2	-15	0.15
30	0.6	50	0.75
60	0.2	100	0.10

a. The mean of Venture 1 is 29 and the mean of Venture 2 is 45.25. Compute the standard deviation for each venture.

Venture 1:

$$\text{Var}(x) = 0.2(-5-29)^2 + 0.6(30-29)^2 + 0.2(60-29)^2 = 424$$

$$\sigma = \sqrt{424} = 20.59$$

Venture 2:

$$\text{Var}(x) = 0.15(-15-45.25)^2 + 0.75(50-45.25)^2 + 0.1(100-45.25)^2 = 433.6875$$

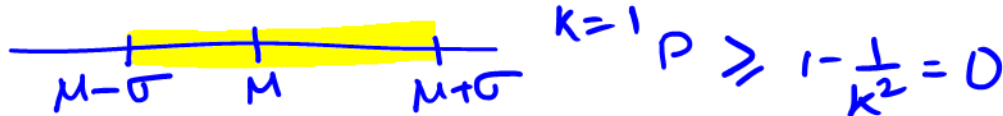
$$\sigma = \sqrt{433.6875} = 20.83$$

b. Which investment would provide the investor with the higher expected return?

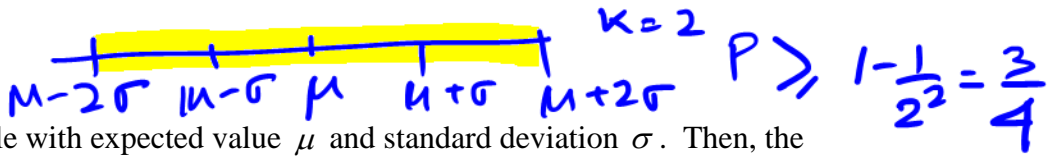
Venture 2 greater mean

c. Which investment would the element of risk be less?

Venture 1 smaller variance



Chebychev's Inequality



Let X be a random variable with expected value μ and standard deviation σ . Then, the probability that a randomly chosen outcome of the experiment lies between $\mu - k\sigma$ and $\mu + k\sigma$ is at least $1 - \frac{1}{k^2}$; that is, $P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - \frac{1}{k^2}$.

Example 4: A probability distribution has a mean 20 and a standard deviation of 3. Use Chebychev's Inequality to estimate the probability that an outcome of the experiment lies between 12 and 28.

$P(12 \leq X \leq 28)$

$\mu = 20 \quad \sigma = 3$

$\mu - k\sigma = 12$ OR $\mu + k\sigma = 28$

$20 - k \cdot 3 = 12$ $20 + k \cdot 3 = 28$

$8 = 3k$ $3k = 8$

$k = 8/3$ $k = 8/3$

$\geq 1 - \frac{1}{k^2}$

$= 1 - \frac{1}{(8/3)^2} = 1 - \frac{9}{64}$

$= \frac{64}{64} - \frac{9}{64} = \frac{55}{64}$

Example 5: The Great Northwestern Lumber Company employs 400 workers in its mills. It has been estimated that X , the random variable measuring the number of mill workers who have industrial accidents during a 1-year period, is distributed with a mean of 30 and a standard deviation of 4. Use Chebychev's Inequality to estimate the probability that the number of workers who will have an industrial accident over a 1-year period is between 20 and 40, inclusive.

$\mu - k\sigma$ $\mu + k\sigma$ $\mu = 30 \quad \sigma = 4$

$P(20 \leq X \leq 40) \geq 1 - \frac{1}{k^2} = 1 - \frac{1}{(\frac{5}{2})^2}$

$20 = \mu - k\sigma$ OR $40 = \mu + k\sigma$

$20 = 30 - 4k$ $40 = 30 + 4k$

$4k = 10$ $4k = 10$

$k = \frac{10}{4} = \frac{5}{2}$ $k = \frac{5}{2}$

$= 1 - \frac{1}{\frac{25}{4}}$

$= 1 - \frac{4}{25}$

$= \frac{25}{25} - \frac{4}{25}$

$= \frac{21}{25} = 0.84$