Section 7.4
The Binomial Distribution

A binomial experiment has the following properties:

1. The number of trials is fixed.
2. There are two outcomes of the experiment: Success, with probability p and Failure, with probability q. Note: $p + q = 1$.
3. The probability of success in each trial is the same.
4. The trials are independent of each other.

Experiments with two outcomes are called **Bernoulli trials** or **Binomial trials**.

Finding the Probability of an Event of a Binomial Experiment
In a binomial experiment in which the probability of success in any trial is p, the probability of exactly x successes in n independent trials is given by $q = 1 - p$

$$P(X = x) = C(n, x)\, p^x q^{n-x}$$

X is called a **binomial random variable** and its probability distribution is called a **binomial probability distribution**.

Example 1: An experiment consists of 10 independent trials where the probability of success is $\frac{5}{8}$. Find each of the following probabilities. $n = 10$ $p = \frac{5}{8}$ $q = 1 - \frac{5}{8} = \frac{3}{8}$

a. The probability of obtaining exactly 5 successes.

$P(X=5) = C(10,5)\left(\frac{5}{8}\right)^5\left(\frac{3}{8}\right)^5 = 0.1782$

b. The probability of obtaining at least 1 success.

1 or 2 or 3 ... 10   0s

$1 - P(X=0) = 1 - C(10,0)\left(\frac{5}{8}\right)^0\left(\frac{3}{8}\right)^{10} = 0.999$

c. $P(X \leq 1)$ [at most 1] 0 or 1 $[a^0 = 1]$

$= P(X=0) + P(X=1)$

$= \underbrace{C(10,0)}_{1}\underbrace{\left(\frac{5}{8}\right)^0}_{1}\left(\frac{3}{8}\right)^{10} + \underbrace{C(10,1)}_{10}\left(\frac{5}{8}\right)^1\left(\frac{3}{8}\right)^9 = 0.00097$

Mean, Variance and Standard Deviation of a Random Variable

If X is a binomial random variable associated with a binomial experiment consisting of n trials with probability of success p, and probability of failure q, then the mean E(X), variance and standard deviation of X are given by applying the following formulas:

$\mu = np$ (mean)

$Var(X) = npq$

$\sigma = \sqrt{Var(X)}$

$= \sqrt{npq}$ (standard deviation)

Example 2: Consider the following binomial experiment. If the probability that a marriage will end in divorce within 20 years after its start is 0.84, what is the probability that out of 6 couples just married, in the next 20 years:

$n = 6$ $p = 0.84$

$q = 0.16$

Success = Divorce

Failure = No Divorce

a. none will be divorced?

$P(X=0) = C(6,0)(0.84)^0(0.16)^6 = 0.000017$

b. all will be divorced?

$P(X=6) = C(6,6)(0.84)^6(0.16)^0 = 0.3513$

c. Find the mean and standard deviation of the experiment.

$\mu = np = 6 \cdot (0.84) = 5.04$

$\sigma = \sqrt{npq} = \sqrt{6 \cdot (0.84)(0.16)} = 0.898$

Example 3: Consider the following binomial experiment. It is estimated that 34% of the general population has blood type A^+. If a sample of 9 people is selected at random, what is the probability that at least 8 of them have blood type A^+?

S = blood A^+

F = not A^+

8 or 9

$P(X=8) + P(X=9)$

$p = 0.34$

$q = 1-p = 0.66$

$= C(9,8)(0.34)^8(0.66)^1 + C(9,9)(0.34)^9(0.66)^0 = 0.0011$

Example 4: The probability of a person contracting influenza on exposure is 62%. In the binomial experiment for a group of 12 people that has been exposed, what is the probability that at most 10 contract influenza?

S = Cont. Inf F = Not Inf

$p = 0.62$ $q = 0.38$

0 or 1 or 2 ... or 10 or 11 or 12

$1 - [P(X=11) + P(X=12)]$

$= 1 - [C(12,11)(0.62)^{11}(0.38) + C(12,12)(0.62)^{12}(0.38)^0]$

$= 0.973$

(*) A coin is tossed 10 times. What is the prob of getting 5 heads.

$$\frac{C(10,5)}{2^{10}}$$

Binomial

S = Heads F = Tails

$p = \frac{1}{2}$ $q = \frac{1}{2}$

$P(X=5)$

$= C(10,5)\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5$

$= C(10,5)\frac{1}{2^5 \cdot 2^5}$

$= \frac{C(10,5)}{2^{10}}$