1. Solve for the unknowns.

$$2\begin{bmatrix} 4 & x \\ y & -1 \\ -4 & 3 \end{bmatrix} - 3\begin{bmatrix} 4 & 2 \\ -6 & -7 \\ z & -2 \end{bmatrix} = -4\begin{bmatrix} 1 & -3 \\ 5 & 5t \\ -4 & -3 \end{bmatrix}$$

$$2x - 6 = 12 \qquad\qquad 2y - 18 = -20$$
$$2x = 18 \qquad\qquad\qquad 2y = 38$$
$$x = 9 \qquad\qquad\qquad\quad y = 19$$

$$-2 + 21 = -20t \qquad\qquad -8 - 3z = 16$$
$$19 = -20t \qquad\qquad\qquad -3z = 24$$
$$-\frac{19}{20} = t \qquad\qquad\qquad\quad z = -8$$

2. Given the linear system of equations. How would you set up using the coefficient matrix to solve the system?

$$4x - 7y = 11$$
$$6x + 9x = -5$$

$$\overset{A}{\begin{bmatrix} 4 & -7 \\ 6 & 9 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 11 \\ -5 \end{bmatrix}}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -7 \\ 6 & 9 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ -5 \end{bmatrix}$$

$$D = 9(4) - 6(-7) = 36 + 42 = 78$$

$$X = \frac{1}{78} \begin{bmatrix} 9 & 7 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \end{bmatrix}$$

3. Find the inverse of the matrix, if possible.

$$\begin{bmatrix} 5 & 3 & -2 \\ 7 & -4 & 8 \\ -3 & 1 & 9 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 5 & 3 & -2 & 1 & 0 & 0 \\ 7 & -4 & 8 & 0 & 1 & 0 \\ -3 & 1 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\left.\begin{array}{ccccccc} -7 & -\frac{21}{5} & \frac{14}{5} & -\frac{7}{5} & 0 & 0 \\ 7 & -4 & 8 & 0 & 1 & 0 \\ \hline 0 & -\frac{41}{5} & \frac{54}{8} & -\frac{7}{5} & 1 & 0 \end{array}\right] R_2$$

$$\downarrow R_1 = \frac{1}{5}R_1$$

$$\left[\begin{array}{ccc|ccc} \boxed{1} & 3/5 & -2/5 & 1/5 & 0 & 0 \\ 7 & -4 & 8 & 0 & 1 & 0 \\ -3 & 1 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\left.\begin{array}{ccccccc} 3 & \frac{9}{5} & -\frac{6}{5} & \frac{3}{5} & 0 & 0 \\ -3 & 1 & 9 & 0 & 0 & 1 \\ \hline 0 & \frac{14}{5} & \frac{39}{5} & \frac{3}{5} & 0 & 1 \end{array}\right] R_3$$

$$\begin{array}{l} R_2 = -7R_1 + R_2 \\ R_3 = 3R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/5 & -2/5 & 1/5 & 0 & 0 \\ 0 & -41/5 & 54/5 & -7/5 & 1 & 0 \\ 0 & 14/5 & 39/5 & 3/5 & 0 & 1 \end{array}\right]$$

$$\left.\begin{array}{ccccccc} 1 & \frac{3}{5} & -\frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & -\frac{2}{5} & \frac{162}{205} & \frac{-21}{205} & \frac{15}{205} & 0 \\ \hline 1 & 0 & \frac{80}{205} & \frac{20}{205} & \frac{15}{205} & 0 \\ 1 & 0 & \frac{46}{41} & \frac{4}{41} & \frac{3}{41} & 0 \end{array}\right] R_1$$

$$\downarrow R_2 = -\frac{5}{41}R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 3/5 & -2/5 & 1/5 & 0 & 0 \\ 0 & \boxed{1} & -54/51 & 7/41 & -5/41 & 0 \\ 0 & 14/5 & 39/5 & 3/5 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_1 = -\frac{3}{5}R_2 + R_1 \\ R_3 = -\frac{14}{5}R_2 + R_3 \end{array}$$

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$$\left. \begin{array}{cccccc} 0 & -\dfrac{14}{5} & \dfrac{756}{205} & -\dfrac{98}{205} & \dfrac{14}{41} & 0 \\[2mm] 0 & \dfrac{14}{5} & \dfrac{39}{5} & 3/5 & 0 & 1 \end{array} \right\} R_3$$

$$\begin{array}{cccccc} 0 & 0 & \dfrac{2355}{205} & \dfrac{25}{205} & \dfrac{14}{41} & 1 \\[2mm] 0 & 0 & \dfrac{471}{41} & 3/41 & \dfrac{14}{41} & 1 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 16/41 & 4/41 & 3/41 & 0 \\ 0 & 1 & -54/41 & 7/41 & -5/41 & 0 \\ 0 & 0 & 471/41 & 5/41 & \dfrac{14}{41} & 1 \end{bmatrix}$$

$$\downarrow R_3 = \frac{41}{471} R_3$$

$$\begin{bmatrix} 1 & 0 & 16/41 & 4/41 & 3/41 & 0 \\ 0 & 1 & -54/41 & 7/41 & -5/41 & 0 \\ 0 & 0 & \boxed{1} & \dfrac{5}{471} & \dfrac{14}{471} & \dfrac{41}{471} \end{bmatrix}$$

$$R_2 = \frac{54}{41} R_3 + R_2$$
$$R_1 = \frac{-16}{41} R_3 + R_1$$

$$\left. \begin{array}{cccccc} 1 & 0 & \dfrac{16}{41} & \dfrac{4}{41} & \dfrac{3}{41} & 0 \\[2mm] 0 & 0 & -\dfrac{16}{41} & \dfrac{-80}{19311} & \dfrac{-224}{1931} & \dfrac{-16}{471} \end{array} \right\} R_1$$
$$\begin{array}{cccccc} 1 & 0 & 0 & \dfrac{44}{471} & \dfrac{29}{471} & \dfrac{-16}{471} \end{array}$$

$$\left. \begin{array}{cccccc} 0 & 1 & -\dfrac{54}{41} & \dfrac{7}{41} & \dfrac{-5}{41} & 0 \\[2mm] 0 & 0 & \dfrac{54}{41} & \dfrac{90}{6437} & \dfrac{252}{6437} & \dfrac{54}{471} \end{array} \right\} R_2$$
$$\begin{array}{cccccc} 0 & 1 & 0 & \dfrac{29}{157} & \dfrac{-13}{157} & \dfrac{18}{157} \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & \dfrac{44}{471} & \dfrac{29}{471} & \dfrac{-16}{471} \\[2mm] 0 & 1 & 0 & \dfrac{29}{157} & \dfrac{-13}{157} & \dfrac{18}{157} \\[2mm] 0 & 0 & 1 & \dfrac{5}{471} & \dfrac{14}{471} & \dfrac{41}{471} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Find the inverse of the matrix, if possible.

$$\begin{bmatrix} 5 & 3 & -2 \\ 7 & -4 & 8 \\ -3 & 1 & 9 \end{bmatrix}$$

✗ (a) $\begin{bmatrix} 9 & -3 & 2 \\ -7 & 4 & -8 \\ 3 & -1 & 5 \end{bmatrix}$

5. $9 + 3 \cdot 7 + -2 \cdot 3$
45 $-21 - 6 = 18 \neq 1$ ✗

✓ (b) $\begin{bmatrix} \frac{-7}{471} & \frac{29}{471} & \frac{-16}{471} \\ \frac{29}{157} & \frac{-13}{157} & \frac{18}{157} \\ \frac{5}{471} & \frac{14}{471} & \frac{41}{471} \end{bmatrix}$

5. $\frac{-7}{471} + 3 \cdot \frac{29}{157} - 2 \cdot \frac{5}{471} = \frac{220}{471} + \frac{87}{157} - \frac{10}{471}$

$= \frac{220}{471} + \frac{261}{471} - \frac{10}{471}$

$= 1$

✗ (c) $\begin{bmatrix} \frac{-7}{471} & \frac{29}{471} & \frac{16}{471} \\ \frac{29}{157} & \frac{-13}{157} & \frac{-18}{157} \\ \frac{-5}{471} & \frac{14}{471} & \frac{-41}{471} \end{bmatrix}$

5. $\frac{-7}{471} + 3 \cdot \frac{29}{157} + \frac{-2 \cdot 5}{471} = \frac{-220}{471} + \frac{87}{471} + \frac{10}{157 \cdot 471}$

$= \frac{-220}{471} + \frac{261}{471} + \frac{10}{471}$

$= \frac{51}{471} \neq 1$

✗ (d) $\begin{bmatrix} \frac{1}{5} & \frac{1}{7} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} & 1 \\ \frac{1}{2} & \frac{1}{8} & \frac{1}{9} \end{bmatrix}$

$5 \cdot \frac{1}{5} + 3 \cdot \frac{1}{3} - 2 \cdot \frac{1}{2}$

$= 1 + 1 - 1 = 1$

$5 \cdot \frac{1}{7} + 3 \cdot \frac{1}{4} - 2 \cdot \frac{1}{8}$

$\frac{5}{7} + \frac{3}{4} - \frac{1}{4} = \frac{5}{7} + \frac{1}{2} \neq 0$