Some More Problems for Test 2

Math-1313

1) Cakes and Pies, a bakery, bought some equipment for $70,000. The scrap value after 6 years will be $10,000, and the equipment will be depreciated linearly.

a) What is the rate of depreciation?

$$m = \frac{10000 - 70000}{6} = -10{,}000$$

Rate of dep = 10,000

b) Give a linear equation that describes the equipment's book value at the end of the t-th year of use, where $0 \le t \le 6$.

$V(t) = mt + \text{initial value} = -10000t + 70000$

c) What will the equipment's book value be at the end of the third year?

$V(3) = -10000(3) + 70000 = \$40{,}000$

d) When will the equipment be worth $35,000?

$35000 = -10000t + 70000$

$-35000 = -10000t$

$t = 3.5$ years

2) Comfy Feet, a shoe manufacturing company, has a fixed cost of $275,000. The manufacturing cost per pair of shoes is $39, and each pair sells for $89.

a) Find the cost function.

$C(x) = 39x + 275000$

b) If the company manufactures 7500 pairs, what is the total cost incurred?

$C(x) = 39x + 275000 = 39(7500) + 275000$

$= \$567{,}500$

c) Find the revenue function.

$R(x) = 89x$

d) If the company sells 15,300 pairs, what is the revenue received?

$R(15300) = 89(15300) = \$1{,}361{,}700$

e) Find the profit function.

$P(x) = R(x) - C(x) = 89x - (39x + 275000)$

$= 50x - 275000$

f) If the company produces and sells 5000 pairs, what is the profit earned or the loss sustained?

$P(5000) = 50(50000) - 275000$

$= -25000$ Loss $\$25{,}000$

g) Find the break-even point.

$P(x) = 0$

$50x - 275000 = 0$

$x = \frac{275000}{50} = 5500$

$R(x) = 89x$

$= 89(5500)$

$= 489500$

Pt $(5500, 489500)$

h) How many shoes must be produced and sold to make a profit of $50,000?

$P(x) = 50x - 275000$

$50000 = 50x - 275000$

$325000 = 50x$

$6500 = x$

Shoes

3) Sauces Galore produces two types of spaghetti sauce: spicy and regular. Each can of spicy sauce requires 36 ounces of tomato mix, 4 ounces of spices and 10 minutes of processing. Each can of regular sauce requires 24 ounces of tomato mix and 20 minutes of processing. The company has at least 720 ounces of tomato mix, no more than 100 ounces of spices and no more than 8 hours of processing time per day. The company will make a profit of $6 for each can of spicy sauce and $3 for each can of regular sauce, and they wish to maximize their profit.
Set up the linear programming problem by writing the objective function as well as the system of constraints. How many cans of spicy and regular spaghetti sauce must be sold per day to maximize their profit? What is the maximum profit? Let x = number of cans of spicy spaghetti sauce and y = number of cans of regular spaghetti sauce.

	x S	y R	
TM	36	24	≥ 720
S	4	0	≤ 100
PT	10min	20mins	$\leq$ 8hurs
Profit	6	3	

$$\text{Max } P = 6x + 3y$$

Subj to
$$\begin{cases} 36x + 24y \geq 720 \\ 4x \leq 100 \\ 10x + 20y \leq 480 \end{cases} \quad x \geq 0, y \geq 0$$

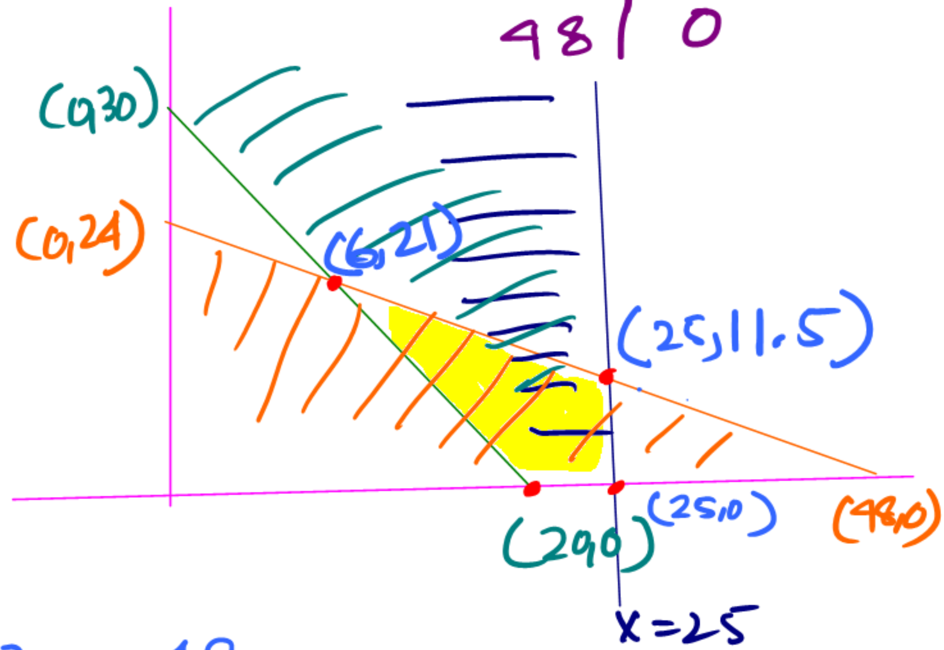
$$\begin{cases} 3x + 2y \geq 60 \\ x \leq 25 \\ x + 2y \leq 48 \end{cases} \quad x \geq 0, y \geq 0$$

$3x + 2y \geqslant 60$ $\qquad$ $x \leq 25$ $\qquad$ $x + 2y \leq 48$

$3x + 2y = 60$ $\qquad$ $x = 25$ $\qquad$ $x + 2y = 48$

x	y
0	30
20	0

x	y
0	24
48	0



$$
\begin{array}{r}
3x + 2y = 60 \\
- \; x + 2y = 48 \\
\hline
2x = 12 \\
x = 6
\end{array}
$$

$x + 2y = 48$

$6 + 2y = 48$

$2y = 42 \quad y = 21$

$x + 2y = 48$

$25 + 2y = 48$

$2y = 23 \quad y = 11.5$

	$P = 6x + 3y$
(6,21)	$36 + 63 = 99$
(20, 0)	120
(25,0)	150
(25, 11.5)	$150 + 34.5 = 184.5 \Leftarrow$

Opt Sol^n $\Big\{$ optimal value = 184.5

pt (25, 11.5)

25 # spicy cans $\qquad$ 11.5 # regular cans

4) Solve the system of linear equations using Gauss-Jordan elimination method

a) $2x + y - 2z = 4$
$x + 3y - z = -3$
$3x + 4y - z = 7$

$$\left[\begin{array}{ccc|c} 2 & 1 & -2 & 4 \\ 1 & 3 & -1 & -3 \\ 3 & 4 & -1 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} \boxed{1} & 3 & -1 & -3 \\ 2 & 1 & -2 & 4 \\ 3 & 4 & -1 & 7 \end{array}\right]$$

$$\begin{array}{cccc} -2 & -6 & 2 & 6 \\ 2 & 1 & -2 & 4 \\ \hline 0 & -5 & 0 & 10 \end{array} \qquad \begin{array}{cccc} -3 & -9 & 3 & 9 \\ 3 & 4 & -1 & 7 \\ \hline 0 & -5 & 2 & 16 \end{array}$$

$R_2 = -2R_1 + R_2$
$R_3 = -3R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & -5 & 0 & 10 \\ 0 & 0 & 2 & 6 \end{array}\right] \xleftarrow{R_3 = -R_2 + R_3} \left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & -5 & 0 & 10 \\ 0 & -5 & 2 & 16 \end{array}\right]$$

$R_3 = \frac{1}{2}R_3$ | $R_2 = -\frac{1}{5}R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & \boxed{1} & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1 = -3R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & \boxed{1} & 3 \end{array}\right]$$

$R_1 = R_3 + R_1$

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$x = 6$
$y = -2$
$z = 3$

unique / one sol^n

b) $3x + y - 6z = -10$
$2x + y - 5z = -8$
$6x - 3y + 3z = 0$

$$\left[\begin{array}{ccc|c} 3 & 1 & -6 & -10 \\ 2 & 1 & -5 & -8 \\ 6 & -3 & 3 & 0 \end{array}\right] \xrightarrow[R_3 = \frac{1}{3}R_3]{R_1 = -R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 2 & 1 & -5 & -8 \\ 2 & -1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 = -R_2 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 2 & 1 & -5 & -8 \\ 0 & -2 & 6 & 8 \end{array}\right] \xrightarrow[R_3 = -\frac{1}{2}R_3]{R_2 = -2R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -3 & -4 \\ 0 & 1 & -3 & -4 \end{array}\right]$$

$$\xrightarrow{R_3 = -R_2 + R_3} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x - z = -2$
$y - 3z = -4$

$x = z - 2$
$y = 3z - 4$

For inf values of z you get inf values of x & y
So the system has inf sol^n

c) $x - 4y - 3z = 0$
$-2x + 9y + 4z = -2$
$3x - 13y - 7z = 2$

$$\left[\begin{array}{ccc|c} 1 & -4 & -3 & 0 \\ -2 & 9 & 4 & -2 \\ 3 & -13 & -7 & 2 \end{array}\right] \xrightarrow[R_3 = -3R_1 + R_3]{R_2 = 2R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -4 & -3 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & 2 & 2 \end{array}\right]$$

$$\begin{array}{cccc} 2 & -8 & -6 & 0 \\ -2 & 9 & 4 & -2 \\ \hline 0 & 1 & -2 & -2 \end{array} \qquad \begin{array}{cccc} -3 & 12 & 9 & 0 \\ 3 & -13 & -7 & 2 \\ \hline 0 & -1 & 2 & 2 \end{array}$$

$R_3 = R_2 + R_3$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -11 & -8 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \xleftarrow{R_1 = 4R_2 + R_1} \left[\begin{array}{ccc|c} 1 & -4 & -3 & 0 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{cccc} 1 & -4 & -3 & 0 \\ 0 & 4 & -8 & -8 \\ \hline 1 & 0 & -11 & -8 \end{array}$$

$$\left.\begin{array}{l} x - 11z = -8 \\ y - 2z = -2 \end{array}\right\}$$

$$\left.\begin{array}{l} x = 11z - 8 \\ y = 2z - 2 \end{array}\right\}$$

Inf many sol^n