11 questions

1 hr

Math 1313 - Chapter 3 review

How to study: Study the class notes, take your practice test, review homework problems and quizzes, and try to do as many exercises as you can from the textbook. Note that answers are provided at the back of the book to all odd numbered problems. Here I provide some examples for you. This is not a complete list, studying only these examples is not enough!

1. Find the transpose of the matrix.

$$\text{(a)} A = \begin{bmatrix} 1 & -4 & 0 \\ 2 & -1 & 2 \\ 0 & -1 & 2 \\ 1 & 3 & 5 \end{bmatrix} \qquad \text{(b)} B = \begin{bmatrix} 2 & -1 & 3 & 5 & -2 & -6 \\ 1 & -1 & 2 & -7 & 3 & 2 \\ 4 & 6 & -3 & 2 & 1 & 0 \\ 0 & 2 & 3 & -2 & 1 & 1 \end{bmatrix}$$

$A^T = \begin{bmatrix} 1 & 2 & 0 & 1 \\ -4 & -1 & -1 & 3 \\ 0 & 2 & 2 & 5 \end{bmatrix}$

$B^T = \begin{bmatrix} 2 & 1 & 4 & 0 \\ -1 & -1 & 6 & 2 \\ 3 & 2 & -3 & 3 \\ 5 & -7 & 2 & -2 \\ -2 & 3 & 1 & 1 \\ -6 & 2 & 0 & 1 \end{bmatrix}$

2. Solve the following.

$$\text{(a)}\ 2\begin{bmatrix} 2 & 0 & -2 \\ -3 & -3 & -4 \\ 1 & -2 & 7 \end{bmatrix} - 4\begin{bmatrix} 5 & -6 & -3 \\ -4 & 2 & 1 \\ 3 & -5 & 6 \end{bmatrix}$$

A_{23} A_{43} A_{12}

$= \begin{bmatrix} 4 & 0 & -4 \\ -6 & -6 & -8 \\ 2 & -4 & 14 \end{bmatrix} - \begin{bmatrix} 20 & -24 & -12 \\ -16 & 8 & 4 \\ 12 & -20 & 24 \end{bmatrix}$

$= \begin{bmatrix} -16 & 24 & 8 \\ 10 & -14 & -12 \\ -10 & 16 & -10 \end{bmatrix}$

$$\text{(b)} \begin{bmatrix} 1 & -1 & 6 & -4 \\ 2 & 3 & -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 7 & 11 \\ 3 & -4 & -2 \\ 1 & 5 & 1 \end{bmatrix}$$

Not possible

$2 \times 4 \quad 3 \times 3$

4x4

(c) $\begin{bmatrix} 2 & -3 \\ 0 & 4 \\ 1 & 7 \\ -5 & -2 \end{bmatrix}_{4\times 2} \cdot \begin{bmatrix} 2 & 7 & 11 & 8 \\ 3 & -4 & -2 & -3 \end{bmatrix}_{2\times 4} = \begin{bmatrix} -5 & 26 & 28 & 25 \\ 12 & -16 & -8 & -12 \\ 23 & -21 & -3 & -13 \\ -16 & -27 & -51 & -34 \end{bmatrix}$

4,2

2,3

3,3

$= \begin{bmatrix} 2\cdot 2+-3\cdot 3 & 2\cdot 7+-3\cdot -4 & 2\cdot 11+-3\cdot -2 & 2\cdot 8+-3\cdot -3 \\ 0\cdot 2+4\cdot 3 & 0\cdot 7+4\cdot -4 & 0\cdot 11+4\cdot -2 & 0\cdot 8+4\cdot -3 \\ 1\cdot 2+7\cdot 3 & 1\cdot 7+7\cdot -4 & 1\cdot 11+7\cdot -2 & 1\cdot 8+7\cdot -3 \\ -5\cdot 2+-2\cdot 3 & -5\cdot 7+-2\cdot -4 & -5\cdot 11+-2\cdot -2 & -5\cdot 8+-2\cdot -3 \end{bmatrix}$

3. Solve for the unknowns.

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ x & -1 \end{bmatrix} - 3\begin{bmatrix} y-1 & 2 \\ 1 & 2 \\ 4 & -3 \end{bmatrix} = 2\begin{bmatrix} -4 & -2 \\ 0 & z \\ 4 & 4 \end{bmatrix}$

$1-3(y-1)=-8$

$-3y+3=-9$

$-3y=-12$

$y=4$

$4-3(2)=2z$

$4-6=2z$

$-2=2z$

$-1=z$

$x-3(4)=2(4)$

$x-12=8$

$x=20$

(b) $5\begin{bmatrix} 4 & x & 7 \\ -3 & 2 & 5 \\ 6 & -4 & s \end{bmatrix} - 2\begin{bmatrix} -2 & -1 & t \\ -y & -3 & -5 \\ 9 & -4 & -6 \end{bmatrix} = 4\begin{bmatrix} 6 & -2 & 7 \\ -9 & 4 & z \\ 3 & w & 2 \end{bmatrix}$

$5x+2=-8$

$5x=-10$

$x=-2$

$-15+2y=-36$

$2y=-21$

$y=\frac{-21}{2}$

$25+10=4z$

$35=4z$

$\frac{35}{4}=z$

$-20+8=4w$

$-12=4w$

$-3=w$

$5s+12=8$

$5s=-4$

$s=-\frac{4}{5}$

$35-2t=28$

$7=2t$

$\frac{7}{2}=t$

Popper #11

1-5 → A

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $D = ad - bc \neq 0$

4. Find the inverse of the matrix, if possible.

(a) $A = \begin{bmatrix} 6 & -9 \\ 8 & -12 \end{bmatrix}$ $D = 6(-12) - 8(-9) = 0$

NO Inverse

(b) $B = \begin{bmatrix} 7 & 11 \\ -9 & -12 \end{bmatrix}$

$D = 7(-12) - 11(-9)$

$= -84 + 99 = 15$

$B^{-1} = \frac{1}{15}\begin{bmatrix} -12 & -11 \\ 9 & 7 \end{bmatrix}$

(c) $C = \begin{bmatrix} -3 & 2 & 4 \\ 7 & -1 & 3 \\ -2 & 3 & 7 \end{bmatrix}$

$\left[\begin{array}{ccc|ccc} -3 & 2 & 4 & 1 & 0 & 0 \\ 7 & -1 & 3 & 0 & 1 & 0 \\ -2 & 3 & 7 & 0 & 0 & 1 \end{array}\right]$ $R_1 = -\frac{1}{3}R_1$ →

$\left[\begin{array}{ccc|ccc} 1 & -2/3 & -4/3 & -1/3 & 0 & 0 \\ 7 & -1 & 3 & 0 & 1 & 0 \\ -2 & 3 & 7 & 0 & 0 & 1 \end{array}\right]$

$R_3 = 2R_1 + R_3$, $R_2 = -7R_1 + R_2$

$-7 \quad 14/3 \quad 28/3 \quad 7/3 \quad 0 \quad 0$

$7 \quad -1 \quad 3 \quad 0 \quad 1 \quad 0$

$0 \quad 11/3 \quad 37/3 \quad 7/3 \quad 1 \quad 0$

$2 \quad -4/3 \quad -8/3 \quad -2/3 \quad 0 \quad 0$

$-2 \quad 3 \quad 7 \quad 0 \quad 0 \quad 1$

$0 \quad 5/3 \quad 13/3 \quad -2/3 \quad 0 \quad 1$

$\left[\begin{array}{ccc|ccc} 1 & -2/3 & -4/3 & -1/3 & 0 & 0 \\ 0 & 11/3 & 37/3 & 7/3 & 1 & 0 \\ 0 & 5/3 & 13/3 & -2/3 & 0 & 1 \end{array}\right]$

$R_2 = \frac{3}{11}R_2$ ↓

$\left[\begin{array}{ccc|ccc} 1 & -2/3 & -4/3 & -1/3 & 0 & 0 \\ 0 & 1 & 37/11 & 7/11 & 3/11 & 0 \\ 0 & 5/3 & 13/3 & -2/3 & 0 & 1 \end{array}\right]$

$R_1 = \frac{2}{3}R_2 + R_1$ ←

$R_3 = -\frac{5}{3}R_2 + R_3$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 10/11 & 1/11 & 2/11 & 0 \\ 0 & 1 & 37/11 & 7/11 & 3/11 & 0 \\ 0 & 0 & -14/11 & -19/11 & -5/11 & 1 \end{array}\right]$

$1 \quad -2/3 \quad -4/3 \quad -1/3 \quad 0 \quad 0$

$0 \quad 2/3 \quad 74/33 \quad 14/33 \quad 2/11 \quad 0$

$1 \quad 0 \quad 30/33 \quad 3/3 \quad 2/11 \quad 0$

$1 \quad 0 \quad 10/11 \quad 1/11 \quad 2/11 \quad 0$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 10/11 & 1/11 & 2/11 & 0 \\ 0 & 1 & 37/11 & 7/11 & 3/11 & 0 \\ 0 & 0 & -14/11 & -19/11 & -5/11 & 1 \end{array}\right]$$

$$\downarrow \quad R_3 = -\frac{11}{14} R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 10/11 & 1/11 & 2/11 & 0 \\ 0 & 1 & 37/11 & 7/11 & 3/11 & 0 \\ 0 & 0 & 1 & 19/14 & 5/14 & -11/14 \end{array}\right]$$

$$\downarrow \quad \begin{aligned} R_2 &= -\tfrac{37}{11} R_3 + R_2 \\ R_1 &= -\tfrac{10}{11} R_3 + R_1 \end{aligned}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -8/7 & -1/7 & 5/7 \\ 0 & 1 & 0 & -55/14 & -13/14 & 37/14 \\ 0 & 0 & 1 & 19/14 & 5/14 & -11/14 \end{array}\right]$$

5. Given the linear system of equations. How would you set up using the coefficient matrix to solve the system?

(a) $3x - 7y = -8$
$4y - 3x = 13$

$3x - 7y = -8$
$-3x + 4y = 13$

$D = 3 \cdot 4 - (-3)(-7)$
$= 12 - 21 = -9 \neq 0$

A X B

$$\begin{bmatrix} 3 & -7 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -7 \\ -3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} -8 \\ 13 \end{bmatrix}$$

$$X = \frac{1}{-9} \begin{bmatrix} 4 & 7 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} -8 \\ 13 \end{bmatrix} = \begin{bmatrix} -4/9 & -7/9 \\ -1/3 & -1/3 \end{bmatrix} \begin{bmatrix} -8 \\ 13 \end{bmatrix}$$

(b) $7x + 2y - 2z = 11$
$-5x + 2y - 4z = -13$
$4x - 3y + 2z = -7$

$$\begin{bmatrix} 7 & 2 & -2 \\ -5 & 2 & -4 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -13 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 & 2 & -2 \\ -5 & 2 & -4 \\ 4 & -3 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 11 \\ -13 \\ -7 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{4}{41} & \frac{-1}{41} & \frac{2}{41} \\ \frac{3}{41} & \frac{-11}{41} & \frac{-19}{41} \\ \frac{-7}{82} & \frac{-29}{82} & \frac{-12}{41} \end{bmatrix} \begin{bmatrix} 11 \\ -13 \\ -7 \end{bmatrix}$$

Try to find the inverse on your own