## Review questions: (try these before class)

1. Find " $x$ " and the measures of $\angle 1$ and $\angle 2$.
$\mathrm{m} \angle 1=7 \mathrm{x}-2$
$\mathrm{m} \angle 2=\mathrm{x}+34$

2. $\overline{\mathrm{WY}}$ bisects $\angle \mathrm{XWZ}$. Find " x ' and $\mathrm{m} \angle \mathrm{YWZ}$ and $\mathrm{m} \angle X W Y$.
$\mathrm{m} \angle \mathrm{YWZ}=9 \mathrm{x}-10$
$\mathrm{m} \angle \mathrm{XWY}=2 \mathrm{x}+39$

3. Find the measure of $\angle 2$.

$$
\begin{aligned}
& \mathrm{m} \angle 1=35^{\circ} \\
& \mathrm{m} \angle 6=52^{\circ}
\end{aligned}
$$


$\mathrm{m} \angle 1=$ $\qquad$
$\mathrm{m} \angle 2=$ $\qquad$
$\mathrm{m} \angle 2=$ $\qquad$
$\mathrm{m} \angle \mathrm{YWZ}=$ $\qquad$
$\mathrm{m} \angle \mathrm{YWZ}=$
$\mathrm{m} \angle \mathrm{XWY}=$ $\qquad$
$x=$ $\qquad$
4. $\mathrm{m} \angle 1=110^{\circ}$
$\mathrm{m} \angle 5=55^{\circ}$
$\mathrm{m} \angle 9=60^{\circ}$


$$
\begin{aligned}
\mathrm{m} \angle 2 & = \\
\mathrm{m} \angle 3 & = \\
\mathrm{m} \angle 4 & = \\
\mathrm{m} \angle 6 & = \\
\mathrm{m} \angle 7 & = \\
\mathrm{m} \angle 8 & =
\end{aligned}
$$

5. $\angle \mathrm{CGF}$ is a right angle, $\mathrm{m} \angle \mathrm{DGE}=30^{\circ}$ and $\mathrm{m} \angle \mathrm{CGD}=\mathrm{m} \angle \mathrm{EGF}=\mathrm{m} \angle \mathrm{AGB}$. Find:
A. $\mathrm{m} \angle \mathrm{AGB}=$ $\qquad$
B. $\mathrm{m} \angle \mathrm{BGC}=$ $\qquad$
C. $\mathrm{m} \angle \mathrm{CGD}=$ $\qquad$
D. $\mathrm{m} \angle \mathrm{EGF}=$ $\qquad$
E. $\mathrm{m} \angle \mathrm{DGF}=$ $\qquad$
F. $m \angle B G E=$ $\qquad$

## Write the hypothesis and the conclusion for the following conditional statements. (try on your own)

1. If two supplementary angles are adjacent, then they are called a Linear Pair.

Hypothesis:

Conclusion:
2. If two angles are complementary, then they add up to $90^{\circ}$.

Hypothesis:

Conclusion:
3. If an angle has a measure less than $90^{\circ}$, then it is called an acute angle.

Hypothesis:

Conclusion:

## The Basic Steps When Writing a Proof:

1. Look at the "given" information - mark the figure with this information.
2. Look at the figure - is there any information you can pull from the figure. (ie: vertical angles, supplementary angles, complementary angles, etc.). This step is important because sometimes our proof statements are right off the figure and not from the "given".
3. Now think about the steps you need to go through to get from the GIVEN statement to the PROVE statement.

MAKE SURE YOU UNDERSTAND THE FOLLOWING CHART

| Reflexive Property | $a=a$ |
| :--- | :--- |
| Symmetric Property | If $a=b$, then $b=a$ |
| Transitive Property | If $a=b$ and $b=c$, then $a=c$ |
| Addition Property | If $a=b$ and $c=d$, then $a+c=b+d$ |
| Subtraction Property | If $a=b$ and $c=d$, then $a-c=b-d$ |
| Multiplication Property | If $a=b$ and $c=d$, then $a c=b d$ |
| Division Property | If $a=b$ and $c \neq 0$, then $\frac{a}{c}=\frac{b}{c}$ |$|$| Substitution Property | If $a=b$, then $a$ and $b$ may be substituted for each other in any <br> $e q u a t i o n ~ o r ~ i n e q u a l i t y . ~$ |
| :--- | :--- |
| Commutative Property | $a+b=b+a, a b=b a$ |
| Associative Property | $a+(b+c)=(a+b)+c, a(b c)=(a b) c$ |
| Distributive Property | $a(b+c)=a b+a c$ |


| Property |
| :--- |
| Segments |
| Reflexive $\mathrm{PQ}=\mathrm{PQ}$ $\mathrm{m} \angle 1=\mathrm{m} \angle 1$ <br> Symmetric $\mathrm{AB}=\mathrm{CD}$ then $\mathrm{CD}=\mathrm{AB}$ $\mathrm{m} \angle \mathrm{A}=\mathrm{m} \angle \mathrm{B}$, then $\mathrm{m} \angle \mathrm{B}=\mathrm{m} \angle \mathrm{A}$ <br> Transitive $\mathrm{GH}=\mathrm{JK}$ and $\mathrm{JK}=\mathrm{LM}$, then $\mathrm{GH}=\mathrm{LM}$ $\mathrm{m} \angle 1=\mathrm{m} \angle 2$ and $\mathrm{m} \angle 2=\mathrm{m} \angle 3$, then $\mathrm{m} \angle 1=\mathrm{m} \angle 3$ |

Determine the appropriate Algebra Property for the following problems. Try These

1. If $2(5 x+3)=16$, then $10 x+6=16$.
2. If $z-5=25$, then $z=30$.
3. $\mathrm{m} \angle 1=\mathrm{m} \angle 1$
4. If $A B=B D$ and $B D=B C$, then $A B=B C$.
5. If $6 x=24$, then $x=4$.

## Essential Parts of the Formal Proof of a Theorem

1. Statement : States the theorem to be proved
2. Drawing: Represents the hypothesis of the theorem.
3. Given: Describes the drawing according to the information found in the hypothesis of the theorem.
4. Prove: Describes the drawing according to the claim made in the conclusion of the theorem.
5. Proof: Orders a list of claims (Statements) and the justification (Reasons), beginning with the given and ending with the Prove, there must be a logical flow in the Proof.

## OK, now wait for me and we will cover the rest in class

## Definition 1:

The converse of a statement "If $P$, then Q " is "If Q , then P ."
That is, the converse of the given statement interchanges the hypothesis and conclusion. The words "if" and "then" do not move.

## Example 1:

Theorem 1.6.1: If two lines are perpendicular, then they meet to form right angles.

Theorem 1.7.1: If two lines meet to form right angles, then these lines are perpendicular.

## Example 2:

Write the converse of the statement:
If a person lives in Houston, then that person lives in Texas.

Theorem 1.7.2: If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.

Theorem 1.7.3: If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.

Theorem 1.7.4: Any two right angles are congruent.

Theorem 1.7.5: If the exterior sides of two acute adjacent angles form perpendicular rays, then these angles are complementary.

## Picture Proof:

## Example 3:

In the figure $B A$
In the figure
$B A \perp B C$. If it is known that $m \angle 1=28^{\circ}$, use this theorem $m$ to find the $m \angle 2$.


Theorem 1.7.6: If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary.

## Example 4:

In the figure $E G$ is a straight line.
a. If it is known that $m \angle 3=128^{\circ}$, find $m \angle 4$.

b. If it known that $m \angle 4=49^{\circ}$, find $m \angle 3$.
c. If it known that $m \angle 3=4 x$ and $m \angle 4=x+20$. Solve for x and find the measures of angles 3 and 4 .

Theorem 1.7.7: If two lines segments are congruent, then their midpoints separate these into four congruent segments.

Theorem 1.7.8: If two angles are congruent, then their bisectors separate these angles into four congruent angles.

## Definition 2:

The negation of a statement makes claim opposite to the original statement. The negation is usually done by using the word "not".

## Definition 3:

The inverse of a conditional statement is formed by negating the hypothesis and negating the conclusion of the original statement. In other words, the word "not" is added to both parts of the sentence.

## Definition 4:

The contrapositive of a conditional statement is formed by negating both the hypothesis and the conclusion, and then interchanging the resulting negations. In other words, the contrapositive negates and switches the parts of the sentence.

| Conditional | $\mathrm{P} \rightarrow \mathrm{Q}$ |  |
| :--- | :--- | :--- |
| Negation | $\sim \mathrm{P}$ |  |
| Converse | $\mathrm{Q} \rightarrow \mathrm{P}$ |  |
| Inverse | $\sim \mathrm{P} \rightarrow \sim \mathrm{Q}$ |  |
| Contrapositive | $\sim \mathrm{Q} \rightarrow \sim \mathrm{P}$ |  |

Fact:
If a conditional statement is true, its contrapositive is TRUE!

## Example 5:

Write the converse, inverse and contrapositive for the following statements.
a. If a polygon is a square, then it has four sides.
b. If $x>2$, then $x \neq 0$
$\mathbf{P} \rightarrow \mathbf{Q}$
P
$\therefore \mathbf{Q}$

$$
\begin{aligned}
& \mathbf{P} \rightarrow \mathbf{Q} \\
& \sim \mathbf{Q} \\
& \\
& \hline \therefore \sim \mathbf{P}
\end{aligned}
$$

## Example 6:

If two angles are vertical angles, then they are congruent.
$\angle 1$ and $\angle 2$ are not congruent.
Conclusion:

Indirect Proofs use the law of negative inference.

## Example 7:

Given: $\quad \angle \mathrm{ABC}$ is not a right angle
Prove: $\angle 1$ and $\angle 2$ are not complementary


## Example 8:

Given: $\quad m \angle 1=m \angle 2$
Prove: $m \angle S X Q=m \angle P X R$


| Statements | x <br> Reasons |
| :--- | :--- |
| 1. $m \angle 1=m \angle 2$ |  |
| 2. $m \angle 1+m \angle 3=m \angle 2+m \angle 3$ |  |
| 3. | $m \angle 1+m \angle 3=m \angle S X Q$ |
|  | $m \angle 2+m \angle 3=m \angle P X R$ |
| 4. $m \angle S X Q=m \angle P X R$ |  |

