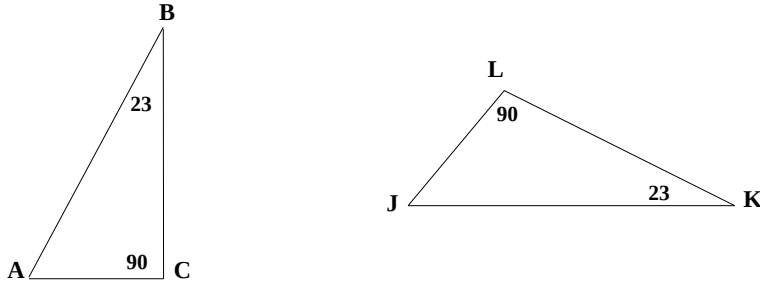


Math 1312 Section 3.1  
Congruent Triangles

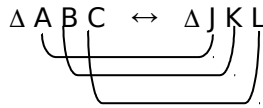
To have a correspondence between two triangles, you must “match up” the angles and sides of one triangle with the angles and sides of the other triangle. Each corresponding angle and side must have the same measure.



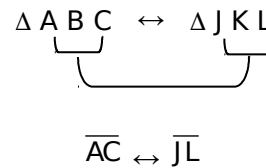
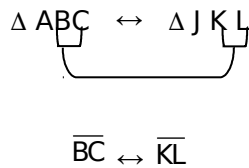
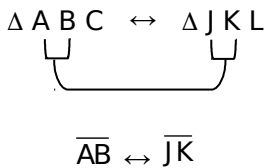
The correspondence between the above two triangles can be stated as  $\triangle ABC \leftrightarrow \triangle JKL$ .

The order in which the letters are written matters since it shows which angles and sides of one triangle match up with the angles and sides of the other triangle:

If  $\triangle ABC \leftrightarrow \triangle JKL$ , the corresponding angles are:



AND the corresponding segments are:



The correspondence may be written in more than one way:  $\triangle CAB \leftrightarrow \triangle LJK$  is the same as  $\triangle ABC \leftrightarrow \triangle JKL$ .

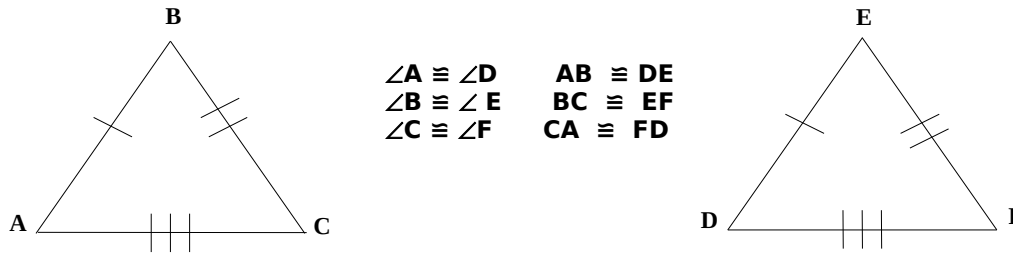
CONGRUENT TRIANGLES

Each triangle has six parts: three sides and three angles. If the six parts of one triangle are congruent to the corresponding six parts of another triangle, then the triangles are *congruent triangles*.

Two triangles are congruent if:

1. All pairs of corresponding angles are congruent.
2. All pairs of corresponding sides are congruent.

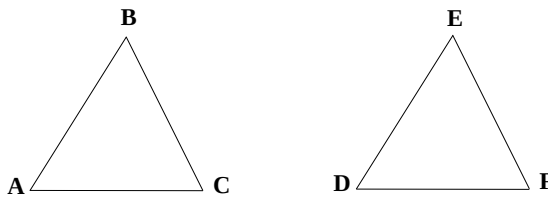
Below,  $\triangle ABC$  and  $\triangle DEF$  are *congruent*, this means that the corresponding parts of each triangle are the same measures. The congruence is written  $\triangle ABC \cong \triangle DEF$ .



Definition of Congruent Triangles (CPCTC) - two triangles are congruent if and only if their corresponding parts are congruent. (CPCTC - corresponding parts of congruent triangles are congruent)

Example 1:

If  $\triangle ABC \cong \triangle DEF$ , name the corresponding congruent angles and sides:

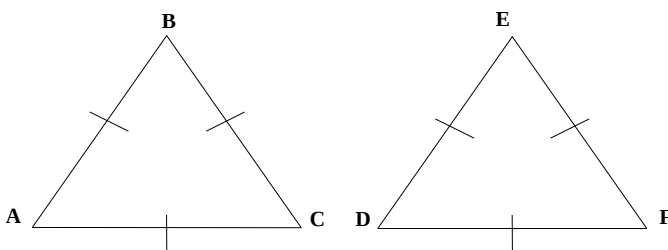


Congruent Angles -

Congruent Sides -

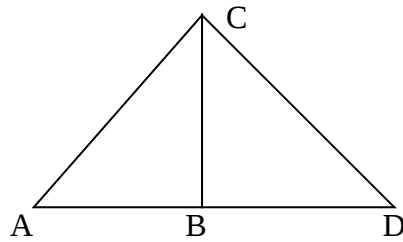
**So the real question. Can we have less information and still show two triangles are congruent?**

(1) SSS Postulate - (side-side-side) if the three sides of one triangle are congruent to the three sides of a second triangle, then the triangles are congruent.



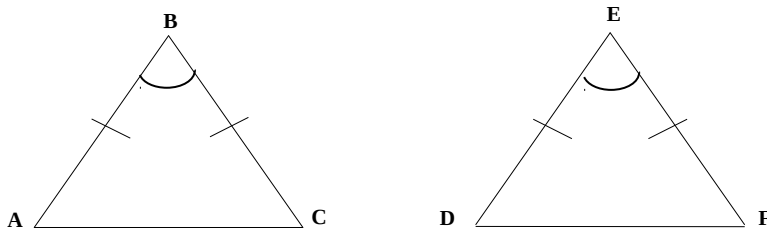
Since all three sides in  $\triangle ABC$  are congruent to all three sides in  $\triangle DEF$ , then  $\triangle ABC \cong \triangle DEF$

Example 2: Here is an example of SSS. If the lines  $AC = DC$  and  $AB = BD$ . Show that  $\triangle ABC \cong \triangle DBC$  using SSS.

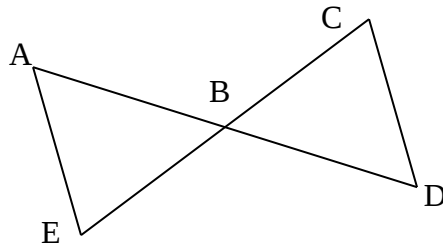


(2) SAS Postulate - if two sides and the “included” angle of one triangle are congruent to two sides and the “included” angle of another triangle, then the triangles are congruent.

In the above definition, the “included” angle is the angle that is formed by the intersection of two sides of a triangle. For example, below  $\angle B$  and  $\angle E$  are the “included” angles.



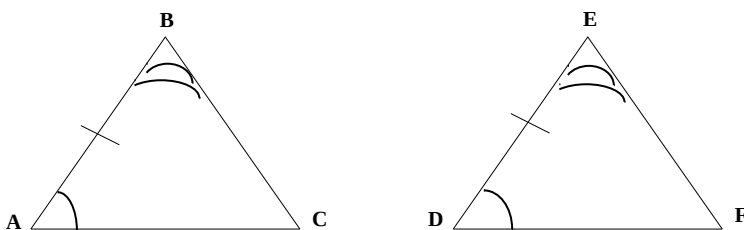
Example 3: Show that  $\triangle ABE \cong \triangle DBC$  using SAS.



B is the midpoint of both  $\overline{AD}$  and  $\overline{EC}$

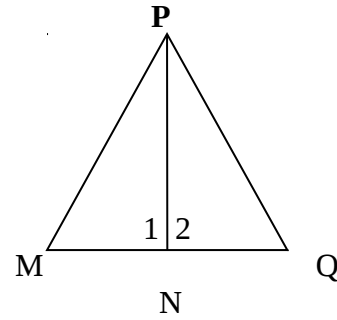
(3) ASA Postulate - if two angles and the “included” side of one triangle are congruent to two angles and the “included” side of another triangle, the triangles are congruent.

In the above definition, the “included” side is the segment that connects two angles. For example, AB and DE are the “included” sides .

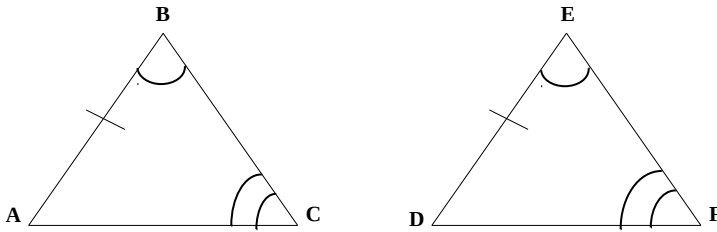


Example 4: Given the following information:

$$\overline{PN} \perp \overline{MQ} ; \angle MPN \cong \angle NPQ$$
$$\triangle PNM \cong \triangle PNQ \text{ by ASA}$$



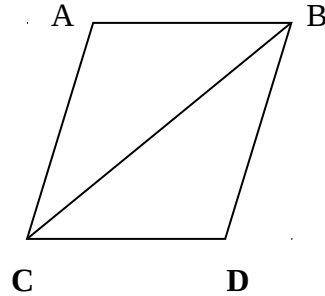
(4) AAS Theorem - if two angles and a “non-included” side of one triangle are congruent to the corresponding two angles and side of a second triangle, the two triangles are congruent.



Example 5: Given the following information:

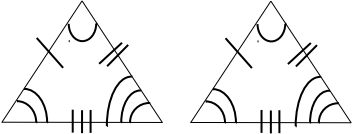
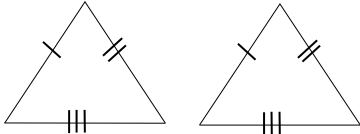
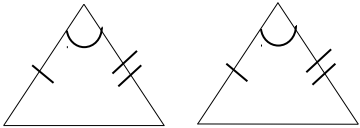
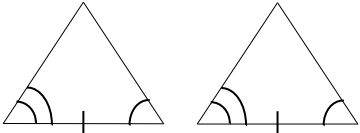
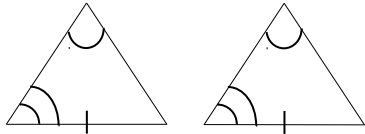
$$\angle A \cong \angle D$$
$$\overline{BC} \text{ bisects both } \angle ACD \text{ and } \angle ABD$$

How can we show congruence using AAS?



**Note: You can not use AAA and SSA because they are not valid for proving triangles are congruent. A triangle can have equal angles can have the same shape but the triangles are not necessarily congruent.**

SUMMARY OF METHODS:

METHOD	QUALIFICATIONS	PICTURE
Def. of $\cong \Delta$	All six parts of one triangle must be congruent with all six parts of the other triangle.	
SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.	
SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.	
ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.	
AAS	Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and non-included side of the other triangle.	

Try these: 3.1 #'s 5, 9 - 12, 13, 19, 21