4.2 The Parallelogram and Kite

Theorems 4.2.1-4.2.3 Have the form of "If _____ then this quadrilateral is a parallelogram."

We will find that quadrilaterals having certain characteristics must be parallelogram.

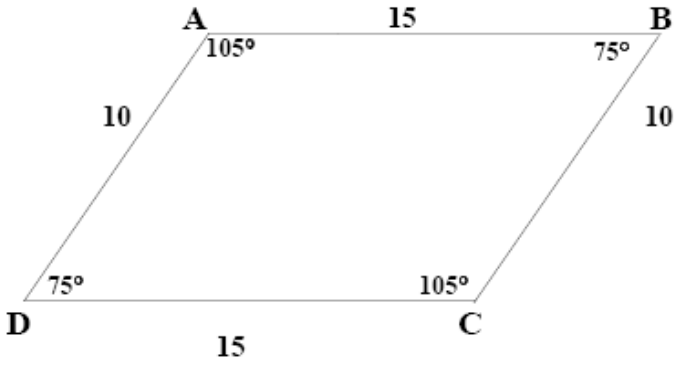
Theorem 4.2.1: If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

Theorem 4.2.2: If both pairs of a opposite sides of a quadrilateral are congruent then it is a parallelogram.

Theorem 4.2.3: If the diagonal of a quadrilateral bisect each other then the quadrilateral is a parallelogram.

Example 1:

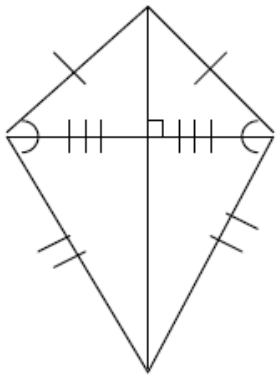
Is ABCD a quadrilateral ? Why? Is ABCD a parallelogram? Why?



Definition: A kite is a quadrilateral with two distinct pairs of congruent angles.

Theorem 4.2.4: In a kite, **one** pair of opposite angles are congruent.

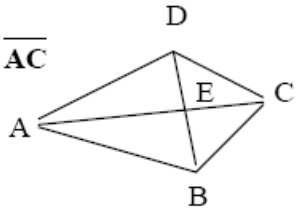
KITES



- Not a parallelogram
- Two pairs of consecutive congruent sides
- The diagonals are perpendicular.
- Exactly one pair of opposite angles are congruent.
- One diagonal is a perpendicular bisector of the other.

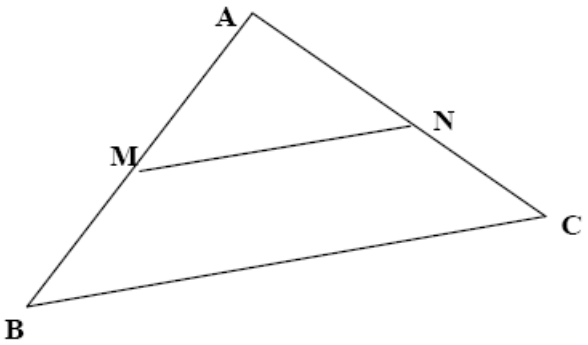
Example 2: Given a kite ABCD, $\overline{\mathbf{AC}}$ is the perpendicular bisector of $\overline{\mathbf{BD}}$:

a. If $\angle \mathbf{B} = \mathbf{90°}$ and AB =8 and BC 6. Find the length of $\overline{\mathbf{AC}}$



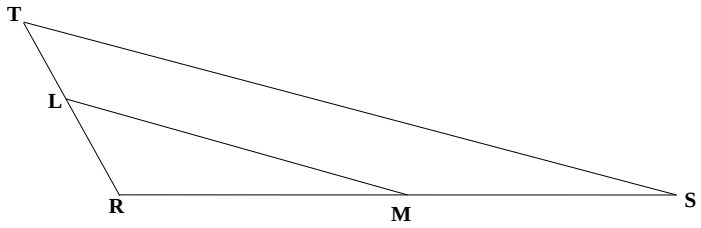
b. If AB = 16.3 and the perimeter of the kite is 54.7, find the lengths of DC, BC and AD.

Theorem 4.2.5: The segment that joins the midpoints of the two sides of a triangle is parallel to the third side and has a length equal to on half the length of the third side.

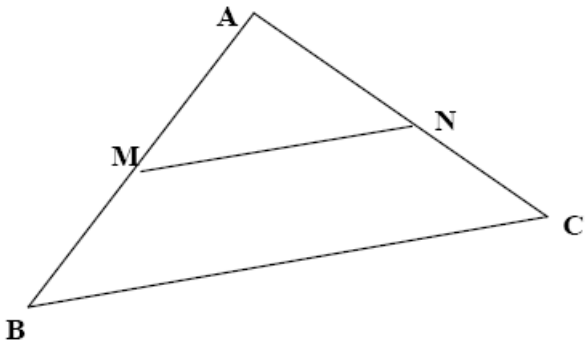


Clarification: in ΔTRS "M" is the midpoint of $\overline{RS}$ and "L" is the midpoint $\overline{RT}$.

By the above "rule", ML ll ST and ML = ½ ST. This can also be expressed as 2ML = ST.



Example 3. M and N are the midpoints of the sides $\overline{AB}$ and $\overline{BC}$ of ΔABC

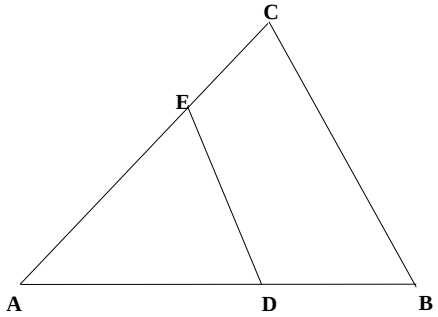


a. If MN = 7.3, find the length of $\overline{BC}$.

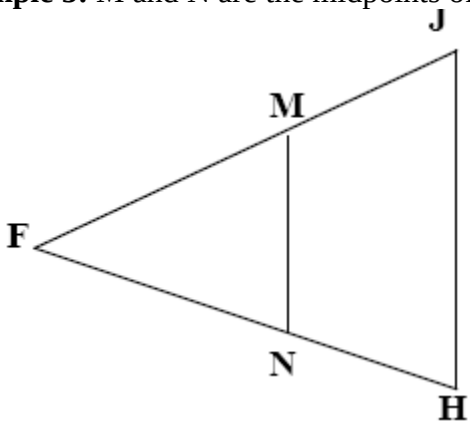
b. If BC = 4x + 6 and MN = x + 9, find the length of $\overline{BC}$.

Example 4:

In the figure below, AE=8, CE=x, DA=6, and BA=12. Is ED ll CB?



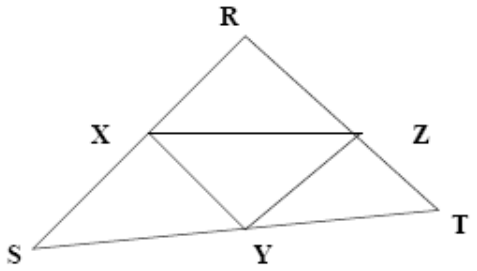
Example 5: M and N are the midpoints of $\overline{FJ}$ and $\overline{FH}$



a. Given that ΔFHJ is isosceles, with $\overline{FJ} \cong \overline{FH}$, FM = 2y +3, NH = 5y−9 and JH = 2y. Find the perimeter of ΔFHJ .

b. Given JH = 12, $m\angle J = 80°$ and $m\angle F = 60°$. Find MN, $m\angle FMN$ and $m\angle FNM$.

Example 6: Use the following figure for both parts a and b. In ΔRST, X, Y and Z are the midpoints of the sides as shown.



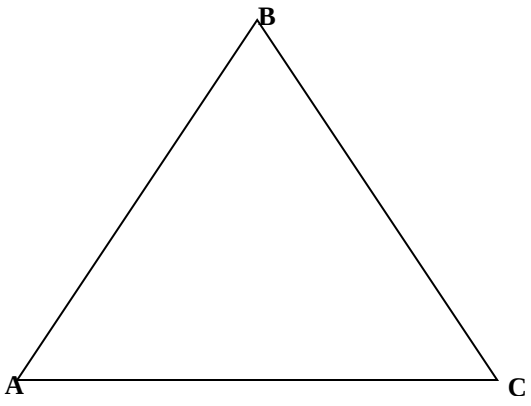
a. **If RS = 18 , RT =24, and ST = 26. Find XY, YZ, XZ and the perimeter of ΔXYZ.**

b. **If XY = 7.2 , XZ = 6.9, YZ = 5.1. Find RS, RT, ST and perimeter ΔRST.**

Example 7:

In ∆ABC, D is the midpoint of AB, E is the midpoint of BC, and F is the midpoint of AC. Find the perimeter of ∆DEF if AB = 24, BC = 32, and AC = 26.

Perimeter of ΔDEF = _____



Example 8: In kite MNPQ. $\overline{MP}$ is the perpendicular bisector of $\overline{NQ}$. If $m\angle QMN = 42°$ and $m\angle MNP = 98°$, find $m\angle NPQ$.