

Ratios, Rates and Proportions

RATIOS:

The ratio of two numbers (a and b) may be written in a variety of ways. For example:

$$\frac{a}{b} \quad a \div b \quad a \text{ to } b \quad a:b$$

In writing the ratio of two numbers, it is usually helpful to express the ratio (fraction) in simplest form. For example, the ratio of 50 to 100 is expressed as follows:

$$\frac{50}{100} = \frac{1}{2}$$

Example 1:

If the measure of $\angle A$ is 60° and $\angle B$ is a right angle, find the ratio of the $m\angle A$ to $m\angle B$.

$$\frac{60}{90} = \frac{2}{3} \text{ (or } 2:3\text{)}$$

$$\frac{100 \text{ miles}}{10 \text{ gallons}} =$$

PROPORTIONS:

An equation that states that two ratios are equal is called a proportion. The following are examples of a proportion:

$$\frac{3}{6} = \frac{1}{2} \qquad \frac{24}{16} = \frac{3}{2}$$

In the above examples, if you cross multiply each proportion you will get the same number on both sides of the equal sign. This is always true with a proportion. A proportion can also contain variables.

Example 2:

$$\frac{3}{6} = \frac{1}{2} \qquad 3 \times 2 = 6 \times 1 \qquad 6 = 6$$

$$\frac{x}{3} = \frac{14}{21} \qquad 21x = 42 \qquad x = 2$$

Property 1 (means-extremes)

In a proportional the product of the means equal the product of the extremes: that is if $\frac{a}{b} = \frac{c}{d}$ (where $b \neq 0$ and $d \neq 0$) then $a \cdot d = c \cdot b$.

Example 3:

Solve for x using Property 1: $\frac{9}{45} = \frac{21}{x}$

Example 4:

Solve for x using Property 1: $\frac{x+5}{9} = \frac{x-1}{3}$

Property 2: In a proportion the means and extremes (or both the means and extremes) maybe interchanged so that if $\frac{a}{b} = \frac{c}{d}$, (where a, b, c, and d are non- zero) then

$$\frac{a}{b} = \frac{c}{d}, \frac{d}{b} = \frac{c}{a}, \text{ and } \frac{d}{c} = \frac{b}{a}$$

Property 3:

If $\frac{a}{b} = \frac{c}{d}$ (where $b \neq 0$ and $d \neq 0$), then $\frac{a+b}{b} = \frac{c+d}{d}$, $\frac{a-b}{b} = \frac{c-d}{d}$

Example 5:

$$1. \frac{6}{8} = \frac{3}{4}$$

$$2. \frac{6}{8} = \frac{3}{4}$$

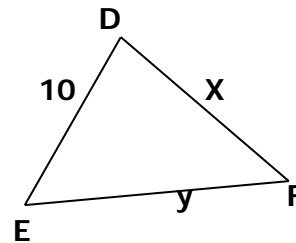
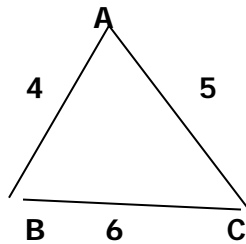
The geometric mean between two positive numbers, a and b , is the positive number, x , where:

$$\frac{a}{x} = \frac{x}{b}$$

Example 6: Find the geometric mean of 4 and 9.

Example 7: find the measure of the three angles of a triangle if the measures of the angles have a ratio 1: 2: 3

Example 8: $\triangle ABC$ and $\triangle DEF$ have the following properties:



Example 9: A recipe calls for 4 eggs and 3 cups of milk. To prepare for a larger number of guests, a cook uses 14 eggs. How many cups of milk are needed?