Math 1312

Section 1.1 : Sets, Statements, and Reasoning

Read ahead and use your textbook to fill in the blanks. We will work the examples together.

A **set** is any collection. These objects are called the elements of the set.

A is a **subset** of *B*, if *A* is "contained" inside *B*. That is, all elements of *A* are also elements of *B*,

in symbols, $A \subseteq B$.

NOTE: *A* and *B* may coincide, i.e. be the same ($A = B$).

Example 1. Examples of sets:

$A = \{1, 2, 3\}$
$B = \{\text{Counting numbers}\}$
$C = \{\text{even numbers less than } 10\} = \{2, 4, 6, 8\}$
D = {Students enrolled in Math 1312}

$\{2\}, \{4\}, \{6\}, \{8\}$
$\{2, 4\}, \{2, 6\}$ - - ..
$\{2, 4, 6\}$ - ..

Set *A* has 3 elements all of which are also the elements of *B*, i.e $A \subseteq B$.

Elements common to *A* and *B* form the **intersection** of A and B, written as $A \cap B$

The **union** of two sets is all elements that are in *A* or *B*, written *as* $A \cup B$

Find: $A \cap B = \{1, 2, 3\}$

Find: $A \cap C = \{2\}$

Find $A \cup C = \{1, 2, 3, 4, 6, 8\}$

$E = \{2, 4, 6\}$
$F = \{1, 3, 5\}$
$E \cup F = \{1, 2, 3, 4, 5, 6\}$
$E \cap F = \{\emptyset\}$

Example 2. You try:

$X = \{3, 19, 5, 7\}$
$Y = \{20, 3, 8, 125, 19\}$

$X \cap Y = \{3, 19\}$

$X \cup Y = \{3, 19, 5, 7, 20, 8, 125\}$

A **statement** is a set of symbols ~~or~~ and words that collectively make a claim that can be classified as true or false.

Example 3. You try and then we can compare answers:

Classify the following as a true statement, false statement, or neither.

5 + 4 = 9
T

5 < 2
F

Triangles have 3 sides.
T

Texas is the largest state in US.
F

Watch out!
Not a statement

An __open____ **statement** is a statement which contains a variable and becomes either true or false depending on the value that replaces the variable.

Example 4

a. x + 2 = 5

True if $x = 3$

b. She is a good tennis player

True if she is Serena Williams

The__negation___ of a statement *P* makes a claim opposite that of the original statement, written as $\sim P$.

Example 5

Statement: All fish can swim.

Negation: some fish cannot swim.

Example 6 (you try)

Write negations for the following statements. Determine the truth value of both, the statement and its negation.

1. Statement: A rectangle has 4 sides.
   Negation: some rectangles does not have 4 sides

P
~P

2. Statement: 2 + 6 = 8
   Negation: $2+6 \neq 8$

$2+6 = 7$ X

3. Statement: $5 \le 2$ F $\quad 5 \ge 2$ T $\quad 5 > 2$ T $\quad 5 < 2$ F
   Negation: $5 > 2$ T $\quad 5 < 2$ F $\quad 5 \le 2$ F $\quad 5 \ge 2$ T

4. Statement: All jokes are funny.
   Negation: Some jokes are not funny

NOTE: A statement and its negation have OPPOSITE truth values!

Construct a **truth table** for the negation of *P*.

P	***~P***
T	F
F	T

We form a **compound statement** by combining simple statements.
Let's use letters *P* and *Q* to represent two simple statements.

- **Conjunction:** ***P*** __and__ ***Q***
- **Disjunction:** ***P*** __or__ ***Q***

A conjunction is TRUE only if BOTH *P* and *Q* are true.

A disjunction is FALSE only if BOTH P and Q are false.

Complete the truth value tables for conjunction and disjunction of *P* and *Q*.

P	***Q***	***P* and *Q***
T	T	T
T	F	F
F	T	F
F	F	F

P	***Q***	***P* or *Q***
T	T	T
T	F	T
F	T	T
F	F	F

Example 7:
Decide if the statement is a Conjunction or Disjunction? Then decide if statement is True or False?

1. Triangles are square or circles are round. Disj , T
   (P: F, Q: T)

2. Triangles are round or circles are square. Disj , F
   (P: F, Q: F)

3. 2 < 1 and 5 < 7   coj , F
   F   T

4. Triangles have 3 angles and 2 + 3 = 5.   conj , T
   F   T

Conditional statement is a compound statement "If *P*, ~~than~~ then *Q*". Here, *P* is called the

hypothesis and *Q* is called the **conclusion**.

"If *P*, then *Q*" can be expressed in the form "All *P* are *Q*".

Example 8

1. If an animal is a fish, then it can swim. ↔ All fish can swim.
   P: H   Q: C

2. If a student is enrolled in this class, then she has to pay the tuition.
   ↔ All students enrolled in this class have to pay the tuition.
   P   Q

Example 9

State the hypothesis and the conclusion.

1. All squares are rectangles.
   H : If a figure is a square
   C : then it is a rectangle

2. You get an "A" in Math 1312 class if you study hard.
   H : If you study hard
   C : then you will get A in Math 1312

Conditional statement is FALSE only if hypothesis is TRUE but conclusion is FALSE.

P	***Q***	**If *P*, then *Q***
T	T	T
T	F	F
F	T	T
F	F	T

Example 10

True or False?

1. If an animal is a fish, then it can swim.   T
   P: T   Q: T

2. 2 + 5 = 7 if triangle has 4 angles.   T
   Q: T   P: F

3. If two lines intersect, then they intersect at a point. T

P Q

4. If Tom studies, then he will get an A on the test.

Not enough information

5. If a number is divisible by 4, then it is divisible by 2. T

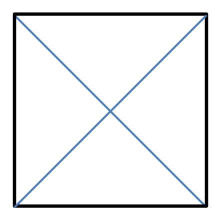
Reasoning is a process based on experience and principles that allow one to arrive at a conclusion.

Types of reasoning

1. **Intuition** – is a way of thinking that draws conclusion from feelings and senses, not based on facts and evidence.
2. **Induction** – is a way of reasoning that draws conclusions from a small number of observations.
3. **Deduction** – is a formal argument that proves the tested theory.

Example 11:

This figure is a square.



1. What can you say about the lengths of the diagonals?

They appear to be equal in length

2. What type of reasoning are you using?

Intution

Example 12:

In a geometry class, you measured the three interior angles of 10 triangles. The triangle s all had 2 equal sides. You discovered that they all have two angles equal in measure.

1. What may you conclude?

If $\triangle$'s have 2 equal sides then $\triangle$'s have 2 equal angles

2. What type of reasoning are you using?

Induction

Example 13.

If a student gets 95 in a test, then he gets an “A”. Tom got 95 in the test.

1. If you accept the above statements as true, what must you conclude?

   Tom will get an 'A'

2. What type of reasoning are you using?

   Deduction

Law of detachment: Let *P* and *Q* represent simple statements and assume that statements 1 and 2 are true. Then a valid argument having conclusion C has the form:

1. If *P*, then *Q*
2. *P*
Conclusion: ∴ Q

(∴ - symbol that means therefore)

- Allows drawing logic conclusions
- Can check if an argument is valid

Example 14

Is the following argument valid?

P Q
1. If it is raining, then Tom will stay at home.
P 2. It is raining
Conclusion: ∴ Tom will stay at home. valid

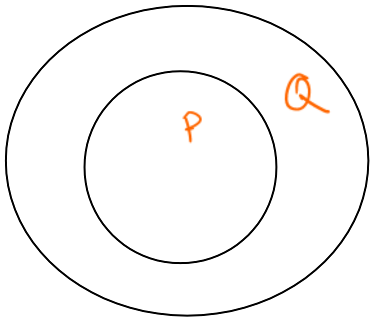
P Q
1. If a man lives in Houston, then he lives in Texas.
2. Mark lives in Texas.
Conclusion: ∴ Mark lives in Houston. Not valid.

Example 15

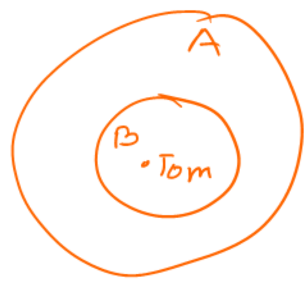
Use deduction to state a conclusion (if possible).

P Q
1. If an angle is a right angle., then it measures 90 °.
P 2. Angle C is a right angle.
Conclusion: ∴ $m\angle C = 90°$

Venn Diagrams
We often use Venn Diagrams to represent sets of objects.
"If P, then Q" can be represented as:



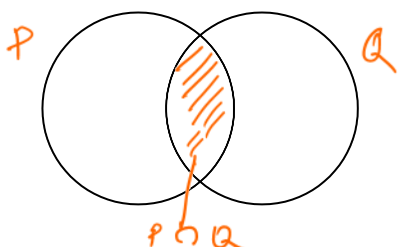
Example 15: *Use Venn Diagram to verify*



1. If a student gets 95 in a test, then he gets an A.
2. Tom got 95 in the test.
Conclusion: Tom got an A in the test.

A = {Students who got an A on the test}
B = {Students who got 95 on the test}

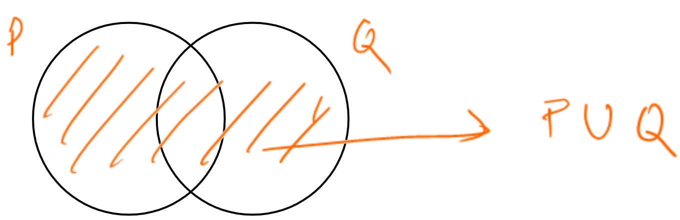
Note: We can also use Venn diagram to represent conjunction and disjunction:



The intersection of P and Q:

$P \cap Q$

The elements that are BOTH in P and in Q



The union of P and Q:

$P \cup Q$

The elements that are in P OR in Q

Use deduction to state a conclusion (if possible)

P Q

1. If a person attends a university, then he will be a success in life.
2. Sam attends University of Houston.

Conclusion:

($\therefore$ Q) Sam will be a success in life

1. If the sum of the measures of two angles is 90^0, then they are called complementary
2. Angle 1 measures 50 degrees and angle 2 measures 40 degrees.

Conclusion:

$\angle 1$ & $\angle 2$ are complementary.

Use Venn diagrams to determine whether the argument is valid or not

1. If an animal is a cat, then it makes "meow" sound.
2. Tom is a cat.

Conclusion: $\therefore$ Tom makes "meow" sound.

Valid

1. If an animal is a cat, then it makes "meow" sound.
2. Tom makes "meow" sound.

Conclusion: $\therefore$ Tom is a cat.

Not Valid

TRY THESE FROM THE TEXTBOOK:

p. 9 # 33 and #35 are practice for using intuition to state a conclusion.

p. 9 # 39 and #40 are practice for using induction to state a conclusion.