Early Definitions and Postulates (1.3)

Four Parts of a Mathematical System

1. Undefined terms points, lines, planes
2. Defined terms Can be defined precisely
3. Axioms or postulates used to prove theorems
4. Theorems prove

Definition: An isosceles triangle is a triangle that has two congruent sides.

Characteristics of a good definition:

1. It names the term being defined.
2. It places the term into a set or category.
3. It distinguishes the defined term from other terms without providing unnecessary facts.
4. It is reversible. If a Δ has two congruent sides it is an isosc Δ.

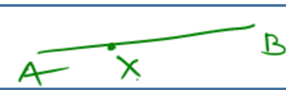
Definition: A line segment is the part of the line that consists of two points, known as endpoints and all points between them.

Postulate 1: Through two distinct points, there is exactly one line.

Postulate 2: The measurement of any line segment is a unique number. (Ruler Postulate)

Definition: The distance between two points A and B is the length of the line segment $\overline{AB}$ that joins the points.

Postulate 3: If X is a point on $\overline{AB}$ and A – X – B then AX + XB = AB



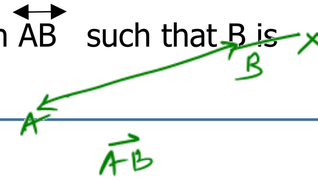
Definition: Congruent (≅) line segments are two lines that have the same length.

Definition: The mid point of a line segment is the point that separates the line segment into two congruent parts.

Example 1: Given M is the midpoint of $\overline{AB}$, AM =3(x+ 3) and MB = 4(x – 2). Find the length of $\overline{AB}$ and the value for x.

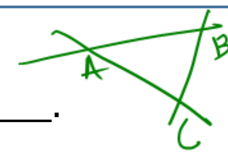
AM = MB (M is the mid pt)
$3(x+3) = 4(x-2)$
$3x + 9 = 4x - 8$
$9 + 8 = 4x - 3x$
$17 = x$
$AM = 3(17+3) = 60$
$\therefore AB = 2(AM)$
$= 2(60)$
$= 120$

Definition: Ray AB denoted by $\overrightarrow{AB}$, is the union of $\overline{AB}$ and all points X on $\overleftrightarrow{AB}$ such that B is between A and X.



Postulate 4: If two lines intersect, they intersect at a ___point___.

Definition: Parallel lines are lines that lie in the same plane but do ___not___ intersect.



Postulate 5: Through three noncollinear points, there is exactly one ___plane___.

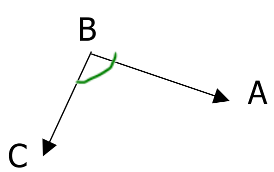
Postulate 6: If two distinct planes intersect, then their intersection is a ___line___.

Postulate 7: Given two distinct points in a plane, the line containing these points also lies in that plane.

Theorem 1.3.1: The midpoint of a line is ___unique___.

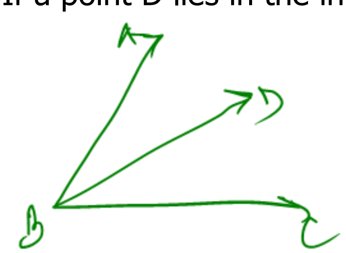
Angles and Their Relationships (1.4)

Definition: An angle is the union of two rays that share a common ___end point___.



Postulate 8: The measurement of an angle is a unique positive number.

Postulate 9: If a point D lies in the interior of an angle ABC, then $\angle ABD + \angle DBC = \angle ABC$

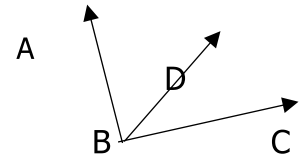


Example 2: Given: $m\angle ABD = 2x + 5$

$m\angle DBC = 3x - 4$

$m\angle ABC = 86°$

Find $m\angle DBC$



$m\angle ABD + m\angle DBC = m\angle ABC$

$2x + 5 + 3x - 4 = 86$

$5x + 1 = 86$

$5x = 85$

$x = 17$

$m\angle DBC = 3(17) - 4$

$= 47°$

$m\angle ABD = 2(17) + 5$

$= 39°$

Definition: Two angles are ___adjacent___ (adj. $\angle s$) if they have a common vertex and a common side between them. (Check-out the last example).

Definition: ___congruent___ angles ($\cong \angle s$) are two angles of the same measure.

Definition: The ___bisector___ of an angle is the ray that separates the given angle into two congruent angles.

Example 3: Given: $\overrightarrow{BD}$ bisects $\angle ABC$

$m\angle$ ABD = x + y

$m\angle DBC = 2x - 2y$

$m\angle ABC = 64°$ find x and y

Definition: ___Vertical___ **Angles -** is where to straight lines intersect, the pairs of nonadjacent angles formed are vertical angles. Vertical angles are congruent. The two adjacent angles are supplementary.

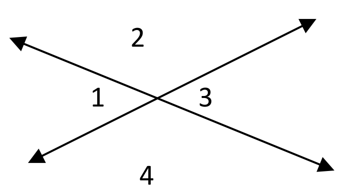
$m\angle 1 = m\angle 3$
$m\angle 2 = m\angle 4$
vertical angles

$m\angle 1 + m\angle 2 = 180°$

$m\angle 2 + m\angle 3 = 180°$

$m\angle 3 + m\angle 4 = 180°$

$m\angle 4 + m\angle 1 = 180°$



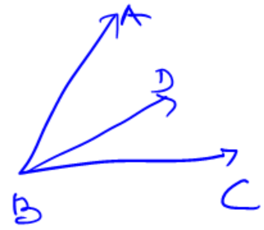
Example 3

$m\angle ABD = x + y$

$m\angle DBC = 2x - 2y$

$m\angle ABC = 64^\circ$

$\overrightarrow{BD}$ is the bisector



$m\angle ABD + m\angle DBC = m\angle ABC$

$x + y + 2x - 2y = 64$

$3x - y = 64$

Since $\overrightarrow{BD}$ is the bisector

$m\angle ABD = m\angle DBC$

$x + y = 2x - 2y$

$3y = x$

$3(3y) - y = 64$

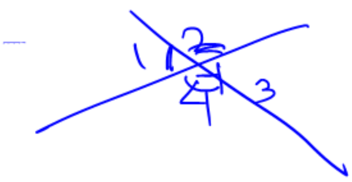
$9y - y = 64$

$8y = 64 \Rightarrow y = 8$

$\therefore \; x = 3y = 3(8) = 24$

$\therefore \; m\angle ABD = x + y = 24 + 8 = 32^\circ$

$m\angle \cancel{A}BC = 2x - 2y = 2(24) - 2(8) = 32^\circ$



Example 4: Use the figure from above.

a. If $m\angle 4 = 97°$, find the measures of the other 3 angles.

$m\angle 4 = m\angle 2 \quad \therefore \quad m\angle 2 = 97°$

$m\angle 1 + m\angle 2 = 180$

$m\angle 1 + 97 = 180$

$m\angle 1 = 180 - 97 = 83°$

$m\angle 3 = m\angle 1$

$\therefore m\angle 3 = 83°$

b. If $m\angle 1 = x + 7$ and the $m\angle 2 = 2x - 23$, find x and the measures of four angles.

$m\angle 1 + m\angle 2 = 180$

$x + 7 + 2x - 23 = 180$

$3x - 16 = 180$

$3x = 196$

$x = \frac{196}{3} = 65.33°$

$m\angle 1 = 65.33 + 7 = 72.33$

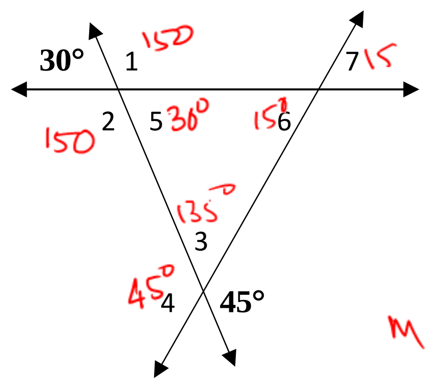
$m\angle 3 = 72.33$ (as $m\angle 1 = m\angle 3$)

$m\angle 2 = 2(65.33) - 23$

$= 107.67°$

$m\angle 4 = 107.67°$

Example 5: Use the figure to answer each questions.



Find the measure of all the angles 1 -7.

$m\angle 1 + 30 = 180$

$m\angle 1 = 150°$

$m\angle 1 = m\angle 2$ (V. A)

$m\angle 3 + 45 = 180$

$m\angle 3 = 180 - 45 = 135°$

$m\angle 6 = m\angle 7$ (V A)

Hint: $\mathbf{m\angle 3 + m\angle 5 + m\angle 6 = 180°}$

$30 + 135 + m\angle 6 = 180 \Rightarrow 165 + m\angle 6 = 180 \Rightarrow m\angle 6 = 15°$

TRY THESE: textbook page 27, #'s 14, 16, 26 and textbook page 35 #'s 10, 18, 23, 26