Introduction to Geometric Proof

Properties of Equality

Addition Property of Equality	If a = b, then a + c = b + c
Subtraction Property of Equality	If a = b, then a – c = b – c
Multiplication Property of Equality	If a = b, then a • c = b • c
Division Property of Equality	If a = b and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

Example 1:

a. If 3x = 9, then x = 3 uses the multiplication or division property

b. If x + 2 = 10, then x = 8 uses the subtraction property

c. If $\frac{2}{3}x = 8$, then x = 12 uses the multiplicand property

Further properties of Algebra

Distributive Property	a(b + c)= a • b + a • c
Substitution Property	If a = b, then a replaces b in any equation.
Transitive Property	If a = b and b = c, then a = c.
Symmetric Property	If $a = b$, then $b = a$.
Reflexive Property	If $a = a$

Example 2: Given: 3x + 2= 4 + 5x Prove: x = − 1

Statements	Reasons
1. 3x +2 = 4 + 5x	1. Given
2. 3x + 2 − 4 = 4 − 4 + 5x	2. subtraction
3. 3x − 2 = 5x	3. Simplification
4. 3x − 3x − 2 = 5x − 3x	4. subtraction
5. − 2 = 2x	5. Simplification
6. $\frac{1}{2}(-2) = \left(\frac{1}{2}\right)2x$	6. Multiplication
7. $-1 = x$	7. simplification
8. $x = -1$	8. symmetric

A 9 B 2 C 9 D

Example 3: Given the drawing

Suppose that AB = 9, BC = 2 and CD = 9 is AC = BD and why?

AB = CD

AB + BC = CD + BC

AC = CD

Transitive property

Example 4:

A C

Given: B is the midpoint of the line $\overleftrightarrow{\mathbf{AC}}$

Prove: AB = $\dfrac{\mathbf{AC}}{\mathbf{2}}$

Statements	Reasons
1. B is the midpoint of $\overline{AC}$	1. Given
2. AB = BC	2. Def of mid pt
3. AB + BC = AC	3. Segment Addition
4. AB + AB = AC	4. Substution
5. 2 (AB) = AC	5. Addition / substitution
6. AB = $\dfrac{AC}{2}$	6. Division / multiplication

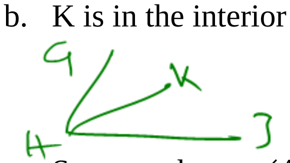
Be sure to study the examples in the book for this section.

Example 5: Answer the following questions.

a. If the $m\angle 1 + m\angle 2 = 90°$ and $m\angle 3 = m\angle 1$ what is true?

$m\angle 3 + m\angle 2 = 90°$ substitution

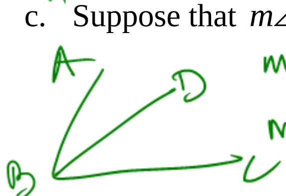
b. K is in the interior of $\angle GHJ$ so what can we conclude about $m\angle GHK + m\angle KHJ =$ $m\angle GHJ$



Angle addition

c. Suppose that $m\angle ABC = 128°$. If $\overline{BD}$ bisects $\angle ABC$, determine the $m\angle ABD$

$m\angle ABD = m\angle DBC$



$m\angle ABD + m\angle DBC = m\angle ABC$

$m\angle ABD =$ $\dfrac{m\angle ABC}{2} = \dfrac{128}{2} = 64°$

$m\angle ABD + m\angle ABD = m\angle ABC$

NOW TRY FROM TEXBOOK: p. 42 #'s 23, 27, 29