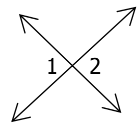
Review questions: (try these before class)

1. Find "x" and the measures of ∠1 and ∠2.

   m∠1 = 7x - 2

   m∠2 = x + 34



x = 6

m∠1 = 40

m∠2 = 40

$m\angle 1 = m\angle 2$

$7x - 2 = x + 34$

$7x - x = 34 + 2$

$6x = 36$

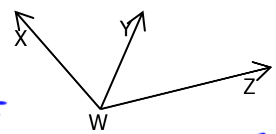
$x = \frac{36}{6} = 6$

$m\angle 1 = 7(6) - 2 = 42 - 2 = 40$

2. $\overline{WY}$ bisects ∠XWZ. Find "x' and m∠YWZ and m∠XWY.

   m∠YWZ = 9x - 10

   m∠XWY = 2x + 39



x = 7

m∠YWZ = 53°

m∠XWY = 53°

$\therefore m\angle YWZ = m\angle XWZ$

$9x - 10 = 2x + 39$

$7x = 49$

$x = 7$

$m\angle YWZ = 9(7) - 10$

$= 63 - 10$

$= 53$

$m\angle XWY$

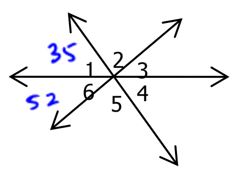
$= 2(7) + 39$

$= 14 + 39 = 53$

3. Find the measure of ∠2.

   m∠1 =35°

   m∠6 = 52°



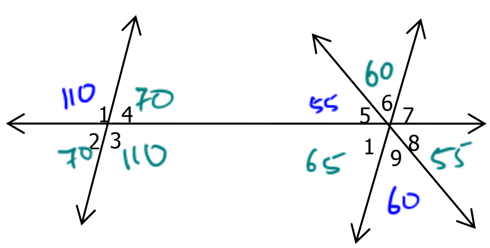
m∠2 = 93°

$m\angle 1 + m\angle 2 + m\angle 6 = 180°$

$35° + m\angle 2 + 52 = 180$

$m\angle 2 = 180 - 35 - 52 = 93°$

4. m∠1 = 110°
   m∠5 = 55°
   m∠9 = 60°



m∠2 = ________

m∠3 = ________

m∠4 = ________

m∠6 = ________

m∠7 = ________

m∠8=________

$m\angle 1 + m\angle 9 + m\angle 8 = 180$

5. ∠CGF is a right angle, m∠DGE = 30° and m∠CGD = m∠EGF = m∠AGB. Find:

A. m∠AGB = 30°

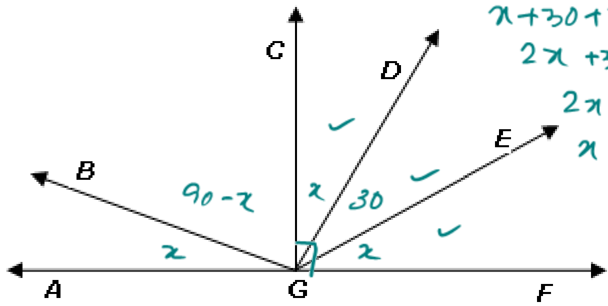
B. m∠BGC = 60°

C. m∠CGD = 30°

D. m∠EGF = 30°

E. m∠DGF = 60°

F. m∠BGE = 120°



$x + 30 + x = 90$

$2x + 30 = 90$

$2x = 60$

$x = 30$

Write the hypothesis and the conclusion for the following conditional statements. (try on your own)

1. If two supplementary angles are adjacent, then they are called a Linear Pair.

p q

Hypothesis: 2 supp ∠s are adj

Conclusion: they are linear pair

2. If two angles are complementary, then they add up to 90°.

Hypothesis:

Conclusion:

3. If an angle has a measure less than 90°, then it is called an acute angle.

Hypothesis:

Conclusion:

The *Basic* Steps When Writing a Proof:

1. Look at the "given" information - mark the figure with this information.

2. Look at the figure - is there any information you can pull from the figure. (ie: vertical angles, supplementary angles, complementary angles, etc.). This step is important because sometimes our proof statements are right off the figure and not from the "given".

3. Now think about the steps you need to go through to get from the **GIVEN** statement to the **PROVE** statement.

MAKE SURE YOU UNDERSTAND THE FOLLOWING CHART

Reflexive Property	$a = a$
Symmetric Property	If $a=b$, then $b=a$
Transitive Property	If $a=b$ and $b=c$, then $a=c$
Addition Property	If $a=b$ and $c=d$, then $a+c = b+d$
Subtraction Property	If $a=b$ and $c=d$, then $a-c = b-d$
Multiplication Property	If $a=b$ and $c=d$, then $ac=bd$
Division Property	If $a=b$ and $c\neq 0$, then $\frac{a}{c} = \frac{b}{c}$
Substitution Property	If $a=b$, then a and b may be substituted for each other in any equation or inequality.
Commutative Property	$a + b = b + a$, $ab = ba$
Associative Property	$a + (b + c) = (a + b) + c$, $a(bc) = (ab)c$
Distributive Property	$a(b + c) = ab + ac$

Property	Segments	Angles
Reflexive	$PQ = PQ$	$m\angle 1 = m\angle 1$
Symmetric	$AB = CD$ then $CD = AB$	$m\angle A = m\angle B$, then $m\angle B = m\angle A$
Transitive	$GH = JK$ and $JK = LM$, then $GH = LM$	$m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$

Determine the appropriate Algebra Property for the following problems. **Try These**

1. If $2(5x + 3) = 16$, then $10x + 6 = 16$. Distributive
2. If $z - 5 = 25$, then $z = 30$. Addition
3. $m\angle 1 = m\angle 1$ Reflexive
4. If $AB=BD$ and $BD=BC$, then $AB=BC$. Transitive
5. If $6x = 24$, then $x = 4$. Division

Essential Parts of the Formal Proof of a Theorem

1. *Statement* : States the theorem to be proved
2. *Drawing*: Represents the hypothesis of the theorem.
3. *Given*: Describes the drawing according to the information found in the hypothesis of the theorem.
4. *Prove*: Describes the drawing according to the claim made in the conclusion of the theorem.
5. *Proof*: Orders a list of claims (Statements) and the justification (Reasons), beginning with the given and ending with the Prove, there must be a logical flow in the Proof.

OK, now wait for me and we will cover the rest in class

Definition 1:
The **converse** of a statement "If P, then Q" is "If Q, then P."
That is, the converse of the given statement interchanges the hypothesis and conclusion.
The words "if" and "then" do not move.

Example 1:
Theorem 1.6.1: If $\underbrace{\text{two lines are perpendicular}}_{P}$, then $\underbrace{\text{they meet to form right angles}}_{Q}$.

Converse
tv

Theorem 1.7.1: If $\underbrace{\text{two lines meet to form right angles}}_{Q}$, then $\underbrace{\text{these lines are perpendicular}}_{P}$.

Example 2:
Write the converse of the statement:
If $\overbrace{\text{a person lives in Houston}}^{P}$, then $\underbrace{\text{that person lives in Texas}}^{Q}$.

If a person lives in Texas, then the person lives in Houston.

Theorem 1.7.2: If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.
suppose $\angle 1$ & $\angle 2$ are comp. to $\angle 3$
$\therefore$ $m\angle 1 + m\angle 3 = 90$
$m\angle 2 + m\angle 3 = 90$
$m\angle 3 = m\angle 3$
$m\angle 1 = m\angle 2$
$\angle 1 \cong \angle 2$

Theorem 1.7.3: If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.

$\angle 1$ & $\angle 2$ supp to $\angle 3$

$m\angle 1 + m\angle 3 = 180$

$m\angle 2 + m\angle 3 = 180$

$m\angle 1 = m\angle 2$

$\angle 1 \cong \angle 2$

Theorem 1.7.4: Any two right angles are congruent.

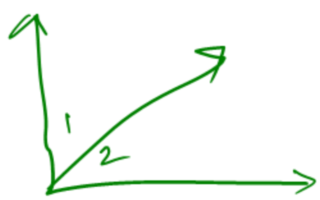
$m\angle 1 = 90$ $\quad m\angle 2 = 90^\circ$

$m\angle 1 = m\angle 2$

$\angle 1 \cong \angle 2$

Theorem 1.7.5: If the exterior sides of two acute adjacent angles form perpendicular rays, then these angles are complementary.

Picture Proof:

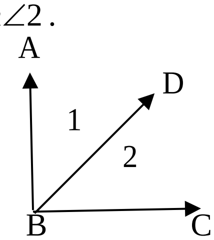


$m\angle 1 + m\angle 2 = 90^\circ$

Example 3:

~~In the figure BA~~

In the figure
$BA \perp BC$. If it is known that $m\angle 1 = 28°$, use this theorem m to find the $m\angle 2$.



$m\angle ABC = 90^\circ$

$\therefore \; m\angle 1 + m\angle 2 = 90$

$28 + m\angle 2 = 90$

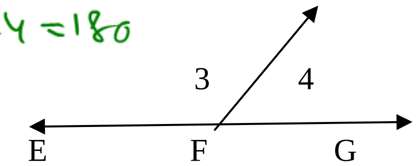
$m\angle 2 = 90 - 28 = 62^\circ$

Theorem 1.7.6: If the exterior sides of two adjacent angles form a straight line, then these angles are supplementary.

Example 4:

$m\angle 3 + m\angle 4 = 180$

In the figure EG is a straight line.



a. If it is known that $m\angle 3 = 128°$, find $m\angle 4$.

$128 + m\angle 4 = 180$

$m\angle 4 = 180 - 128 = 52°$

b. If it known that $m\angle 4 = 49°$, find $m\angle 3$.

$m\angle 3 = 180 - 49 = 131°$

c. If it known that $m\angle 3 = 4x$and $m\angle 4 = x + 20$. Solve for x and find the measures of angles 3 and 4.

$4x + x + 20 = 180$

$5x + 20 = 180$

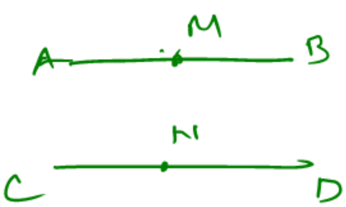
$5x = 160$

$x = \frac{160}{5} = 32°$

$m\angle 3 = 4(32) = 128°$

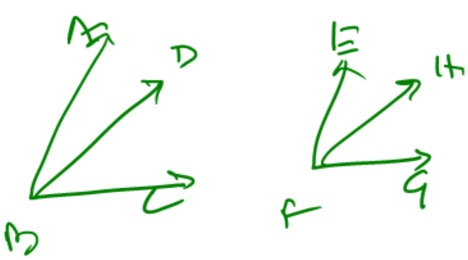
$m\angle 4 = 32 + 20 = 52°$

Theorem 1.7.7: If two lines segments are congruent, then their midpoints separate these into four congruent segments.



$\overline{AM} \cong \overline{MB} \cong \overline{CN} \cong \overline{ND}$

Theorem 1.7.8: If two angles are congruent, then their bisectors separate these angles into four congruent angles.



$\angle ABD \cong \angle DBC \cong \angle EFH \cong \angle HFG$

Definition 2:

The **negation** of a statement makes claim opposite to the original statement. The negation is usually done by using the word "not".

Definition 3:
The **inverse** of a conditional statement is formed by *negating* the hypothesis and *negating* the conclusion of the original statement. In other words, the word "not" is added to both parts of the sentence.

Definition 4:
The **contrapositive** of a conditional statement is formed by *negating* both the hypothesis and the conclusion, ***and*** then *interchanging* the resulting negations. In other words, the contrapositive negates and switches the parts of the sentence.

Conditional	$P \rightarrow Q$	If P, then Q
Negation	$\sim P$	Not P
Converse	$Q \rightarrow P$	If Q, then P
Inverse	$\sim P \rightarrow \sim Q$	If not P, then not Q
Contrapositive	$\sim Q \rightarrow \sim P$	If not Q, then not P

Fact:
If a conditional statement is true, its contrapositive is TRUE!

Example 5:
Write the converse, inverse and contrapositive for the following statements.

a. If $\overbrace{\text{a polygon is a square}}^{P}$, then $\overbrace{\text{it has four sides}}^{Q}$.

Converse: If a polygon has 4 sides, then the poly. is a square

Inverse: If a ploygon is not a square, then it does not have 4 sides.

Contrapositive: If a poly. does not have 4 sides, then it is not a square.

b. If $x>2$, then $x \neq 0$

P: $x > 2$ ~P: $x \leq 2$

Q: $x \neq 0$ ~Q: $x = 0$

Conv.: If $x \neq 0$, then $x > 2$

Inv.: If $x \leq 2$, then $x = 0$

Contra.: If $x = 0$, then $x \leq 2$

The Law of Detachment

If P then Q

$P \to Q$
P

$\therefore Q$

The Law of Negative Inference.

If P, then Q

$P \to Q$ If P, then Q
$\sim Q$ Not Q

$\therefore \sim P$ Not P

Example 6:

1) If two angles are vertical angles (P), then they are congruent (Q).
2) $\angle 1$ and $\angle 2$ are not congruent. ~Q

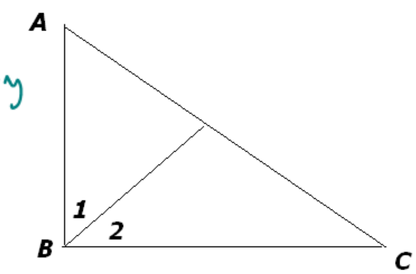
Conclusion:

~P $\angle 1$ & $\angle 2$ are not vertical angles

Indirect Proofs use the law of negative inference.

Example 7:

Given: $\angle$ABC is not a right angle
Prove: $\angle 1$ and $\angle 2$ are not complementary



If $\angle$ABC is not a right angle then $\angle 1$ & $\angle 2$ are not complementary

Contrapositive

If $\angle 1$ & $\angle 2$ are complementary then $\angle$ABC is a right angle

$m\angle 1 + m\angle 2 = 90°$ (def of comp $\angle$s)

$m\angle 1 + m\angle 2 = m\angle ABC$ (angle addition pos)

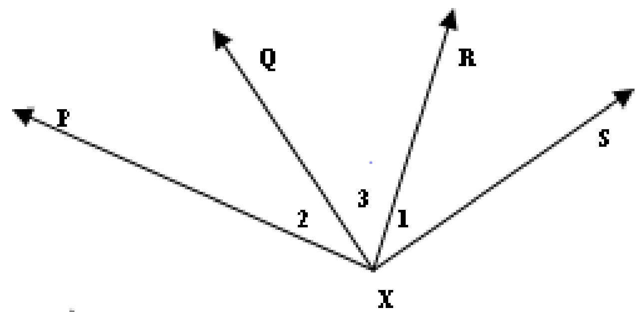
$\therefore m\angle ABC = 90°$ (transitive)

$\therefore$ $\angle$ABC is a right angle

Example 8:

Given: $m\angle 1 = m\angle 2$

Prove: $m\angle SXQ = m\angle PXR$



Statements	Reasons
1. $m\angle 1 = m\angle 2$	Given
2. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	Addition prop of eq^{n}.
3. $m\angle 1 + m\angle 3 = m\angle SXQ$ $m\angle 2 + m\angle 3 = m\angle PXR$	Angle addition postulate
4. $m\angle SXQ = m\angle PXR$	Transitive