Indirect Proof

(either recall these rules or find in 2.2 to fill in the table below)

Conditional	$P \to Q$	If P, then Q
Negation	$\sim P$	Not P
Converse	$Q \to P$	If Q, then P
Inverse	$\sim P \to \sim Q$	If not P, then not Q
Contrapositive	$\sim Q \to \sim P$	If not Q, then not P

Here is a conditional statement:

P Q

If two sides of a triangle are equal, then the triangle is isosceles.

Converse: $Q \to P$

If the triangle is isosceles, then the triangle has two equal sides.

Inverse: $\sim P \to \sim Q$

If the triangle does not have two equal sides, then the triangle is not isosceles.

Contra-positive: $\sim Q \to \sim P$

If a triangle is not isosceles, then the triangle does not have two equal sides.

Example 1:

Write the inverse, converse, and contra-positive of the following statement.

If a number is positive, then the number is greater than zero.

Converse: $Q \to P$ If a number is greater than zero, then the number is positive

Inverse: $\sim P \to \sim Q$ If a number is not positive, then the number is not greater than zero

contrapositive: $\sim Q \to \sim P$ If a no. is not greater than zero, then the no. is not positive.

Be sure to look over example 1 in 2.2 page 77. Copy and place in your notes ☺

Law of Negative Inference (Contra-positive)
This will help so you will understand why contra-positive works.

$$\begin{array}{l} P \rightarrow Q \\ \underline{\sim Q} \\ \sim P \end{array}$$

If P, then Q
→ 1st, ~Q
→ conclusion, ~P

If Pablo lives in Guadalajara, the he lives in Mexico.
Pablo does not live in Mexico.

Conclusion is that: Pablo does not live in Guadalajara.

The Law of Negative Inference is referred to as Indirect Proof.

Example 2:

Assuming that statements 1 and 2 are true, draw a valid conclusion.

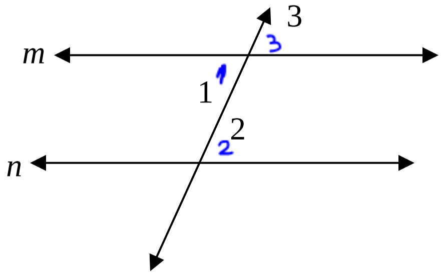
1. If $\underbrace{\text{two angles are both right angles}}_{P}$, then $\underbrace{\text{the angles are congruent}}_{Q}$.

2. $\angle R$ and $\angle S$ are not congruent. ~Q

∴ ~P
Then ∠R & ∠S are not rt. ∠s

Example 3: We are now going to look at a proof done first ~~in~~direct and then indirect.

Given : $\angle 1 \cong \angle 2$
Prove: $m \parallel n$



Direct Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 1 \cong \angle 3$	2. Vertical Angles
3. $\angle 3 \cong \angle 2$	3. Transitive
4. $m \parallel n$	4. Corresponding $\angle$s are $\cong$ then the lines are $\parallel$.

Indirect proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. m is not parallel to n	2. Assumption
3. $\angle 1 \neq \angle 2$	3. Alternate interior $\angle$s are not $\cong$, then lines are not $\parallel$

Contradiction to the given condition
Hence our assumption was wrong

Example 5: If $\angle 1 \neq \angle 2$, then $\angle 1$ and $\angle 2$ are not vertical angles. Indirect proof

P Given: $\angle 1 \neq \angle 2$
Q Prove: $\angle 1$ and $\angle 2$ are not vertical angles.

Statements	Reasons
NQ 1. Suppose $\angle 1$ and $\angle 2$ are vertical angles.	1. Assumption
2. $\angle 1 \cong \angle 2$	2. vertical angles are $\cong$
3. $\angle 1$ and $\angle 2$ are not vertical angles.	3. Contradiction to the given assumption $\angle 1 \neq \angle 2$ so $\angle 1$ & $\angle 2$ are not vertical.

OK, TRY THESE: p. 81 #'s 19, 24