

Proving Parallel Lines  
2.3

**State Theorems 2.1.2 through 2.1.4 and Postulate 11 :**

All of the above theorems start with start with the hypothesis “If two parallel lines are cut by a transversal”

Postulate 11:

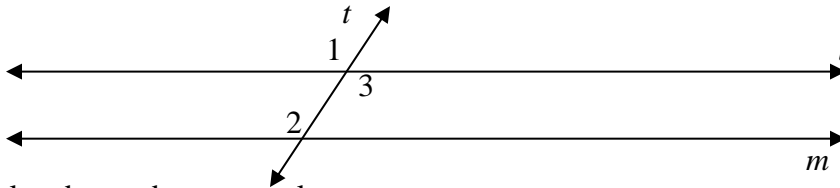
Theorem 2.1.2

Theorem 2.1.3

Theorem 2.1.4

Theorem 2.1.5

**Theorem 2.3.2:** If two lines are cut by a transversal so that interior angles are congruent, then these lines are parallel.



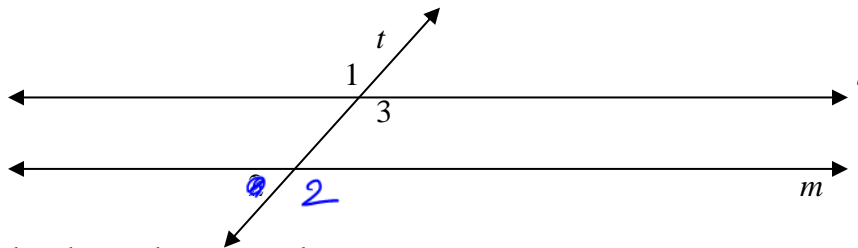
Given: lines  $l$  and  $m$  and transversal  $t$

$$\angle 2 \cong \angle 3$$

Prove:  $l \parallel m$

Statements	Reasons
1. lines $l$ and $m$ and transversal $t$	1. Given
$\angle 2 \cong \angle 3$	2. If two lines intersect then vertical angles are $\cong$
2. $\angle 1 \cong \angle 3$ then vertical angles	3. Transitive property
3. $\angle 1 \cong \angle 2$	4. If two lines are cut by a transversal so that corres. angles are $\cong$ so lines are parallel
4. $l \parallel m$	

**Theorem 2.3.3:** If two lines are cut by a transversal so that alternate exterior angles are congruent



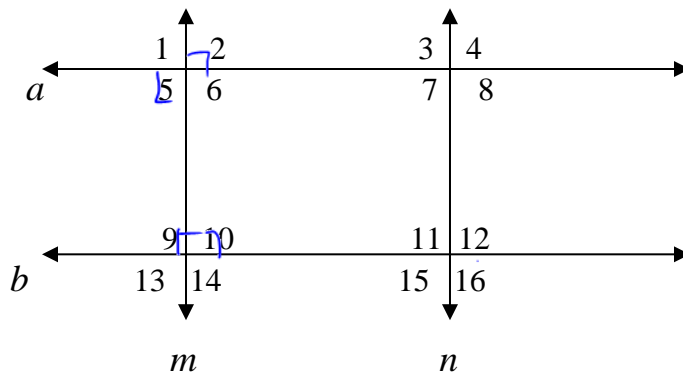
Given: lines  $l$  and  $m$  and transversal  $t$

$$\angle 1 \cong \angle 2$$

Prove:  $l \parallel m$

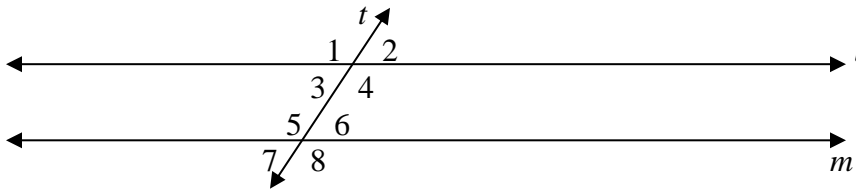
Statements	Reasons
1. lines $l$ and $m$ and transversal $t$ $\angle 1 \cong \angle 2$	1. Given
2. $\angle 1 \cong \angle 3$	2. vertical angles are $\cong$
3. $\angle 2 \cong \angle 3$	3. Transitive
4. $l \parallel m$	4. Corresponding angles are $\cong$ then $l \parallel m$

**Example 1:** Name the lines (if any) that must be parallel under the given conditions.



- a.  $\angle 1 \cong \angle 3$   $m \parallel n$ , corresponding  $\angle$ s
- b.  $\angle 4 \cong \angle 15$   $a \parallel b$ , Alternate exterior
- c.  $\angle 10 \cong \angle 13$  Vertical  $\angle$ s, but we can say anything about  $\parallel$  lines
- d.  $a \perp m$  and  $b \perp m$   $a \parallel b$ , Thm 2.3.7 Line that are  $\perp$  to the same line are  $\parallel$

**Example 2:** Determine the values of  $x$  or the angle so that the line  $l$  will be parallel to  $m$ .



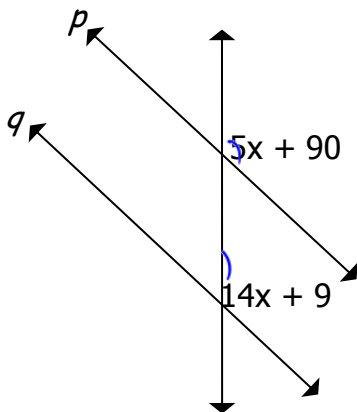
a. If  $m\angle 1 = 107^\circ$ , find  $m\angle 5 = 107$  Corresponding  $\angle$ s

b. If  $m\angle 4 = 106^\circ$ , find  $m\angle 6$   
 $m\angle 4 + m\angle 6 = 180$   
 $m\angle 6 = 180 - 106 = 74^\circ$

c. If  $m\angle 2 = 72^\circ$ , find  $m\angle 7 = 4x + 20$  At  $8x + 20$   
 $m\angle 2 = m\angle 7$   
 $72 = 4x + 20 \Rightarrow 52 = 4x \Rightarrow 13 = x$   
 Find  $x$ ?

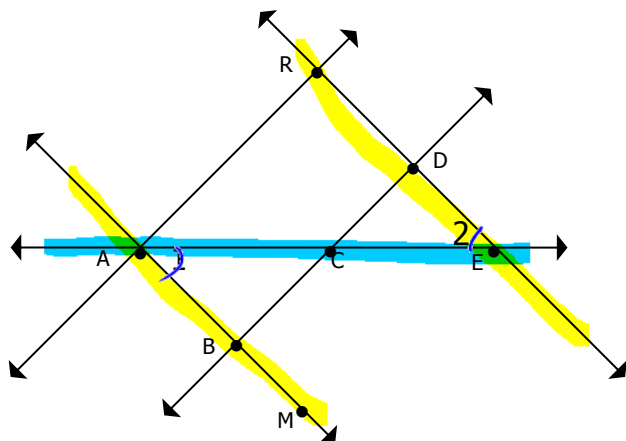
d. If  $m\angle 3 = 2x + 26$ ,  $m\angle 5 = 6(x - 1)$   
 $m\angle 3 + m\angle 5 = 180$   
 $2x + 26 + 6(x - 1) = 180$   
 $2x + 26 + 6x - 6 = 180$   
 $8x + 20 = 180$   
 $8x = 160$   
 $x = 20$   
 $m\angle 3 = 2(20) + 26 = 66^\circ$   
 $m\angle 5 = 6(20 - 1) = 6(19) = 114^\circ$

**Example 3:** Find the value of  $x$  and the measure of each angle that will make  $p \parallel q$



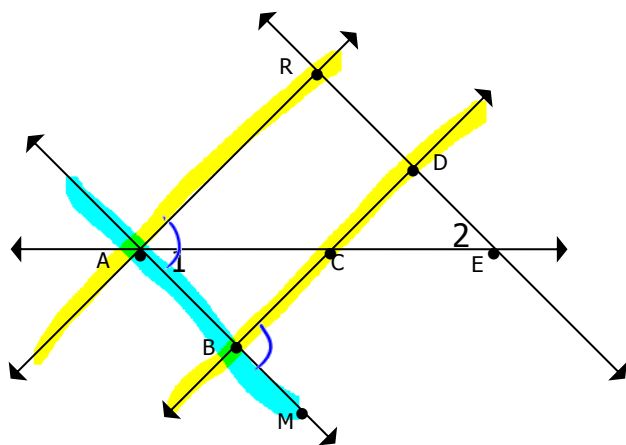
For  $p \parallel q$  we need  
 $5x + 90 = 14x + 9$   
 $90 - 9 = 14x - 5x$   
 $81 = 9x$   
 $9 = x$

**Example 4:** a. If  $\angle 1 \cong \angle 2$  which lines must be parallel?



$\overleftrightarrow{AE} \rightarrow$  transversal  
 $\angle 1 \cong \angle 2$  alt interior  $\angle$ s  
 $\therefore \overleftrightarrow{AM} \parallel \overleftrightarrow{RE}$

b. If  $\angle RAB \cong \angle CBM$ , which lines must be parallel?



$\overleftrightarrow{AM} \rightarrow$  transversal  
 $\angle RAB \cong \angle CBM$  corresponding  $\angle$ s  
 $\therefore \overleftrightarrow{AR} \parallel \overleftrightarrow{BD}$

**OK, TRY:** p. 87 #s 19 and 21