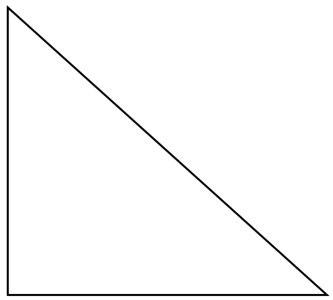
The Angles of a Triangle

Definition: A triangle is the union of three line segments that are determined by three non-collinear points.

Parts of a triangle:

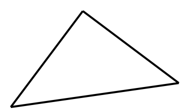


Types of triangles: [fill in the blanks]

Classified by Congruent sides

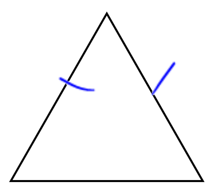
________________ No congruent sides

Scalene



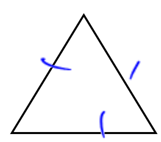
________________ Two congruent sides

Isosceles



________________ Three congruent sides

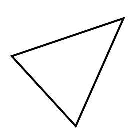
Equilateral



Classify by Angles:

________________ all angles are acute

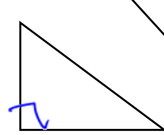
Acute $\angle$ed $\triangle$



all $\angle$s less than 90^0

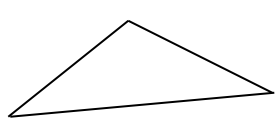
________________ one right angle

Rt. $\angle$ed $\triangle$



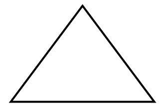
________________ one obtuse angle

Obtuse $\angle$ed $\triangle$



one $\angle$ is more than 90^0

_______________ all angles are congruent



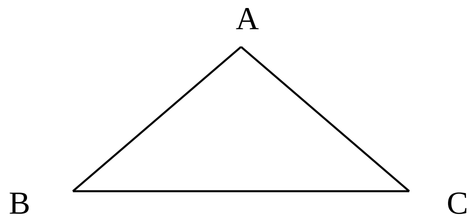
all ∠s are equal

Equiangular

Theorem 2.4.1: In a triangle, the sum measure of the interior angles is $180°$.

Given: ΔABC
Prove: $m\angle A + m\angle B + m\angle C = 180°$

Picture Proof:



Example 1: ΔABC, has $m\angle A = m\angle C = 76°$, find $m\angle B$. What kind of triangle is ΔABC?

$$m\angle A + m\angle B + m\angle C = 180$$
$$76 + m\angle B + 76 = 180$$
$$m\angle B = 180 - 152$$
$$= 28°$$

Isosceles and Acute △

Corollary 2.4.2: Each angle of an equiangular triangle measures $60°$.

Corollary 2.4.3: The acute angles of a right triangle are complementary.

$$m\angle 1 + 90 + m\angle 2 = 180$$
$$m\angle 1 + m\angle 2 = 90$$

Example 2: Classify the triangle form the given information. We will use ΔABC .

a. $m\angle B = 115°$ Obtuse ∠ed Δ

b. $m\angle A = m\angle B = m\angle C$ Equiangular

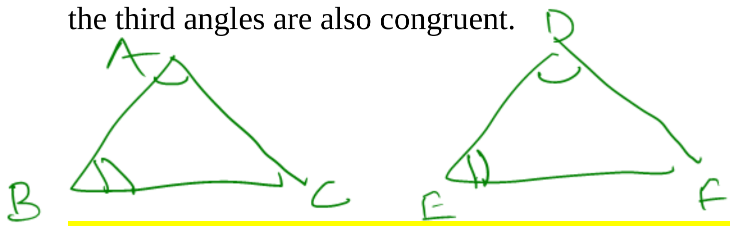
c. $m\angle A = 45°$, $m\angle B = 65°$, $m\angle C = 70°$ Acuted ∠ed Δ

d. $\angle A$ and $\angle C$ are complementary. Right ∠ed Δ

$m\angle A + m\angle B + m\angle C = 180$ $\quad m\angle A + m\angle C = 90$

$m\angle B + 9 = 180 \Rightarrow m\angle B = 90$

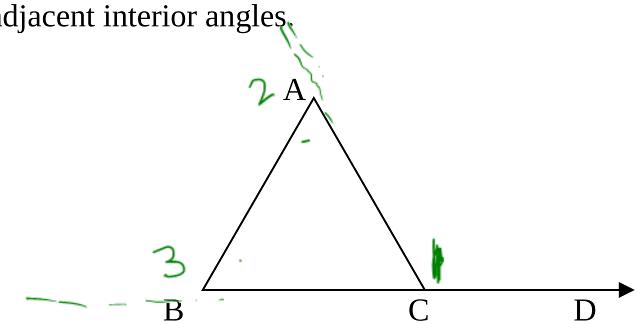
Corollary 2.4.4: If two angles of one triangle are congruent to two angles of another triangle then the third angles are also congruent.



$\left.\begin{array}{l}\text{If } \angle A \cong \angle D \\ \angle B \cong \angle E\end{array}\right\} \Rightarrow \angle C \cong \angle F$

Look over example 4, p. 91. It is a good example and we will use this corollary will be used in chapter 3.

Corollary 2.4.5: The measure of an exterior angle of a triangle equals the sum of the two measures of the two non adjacent interior angles.

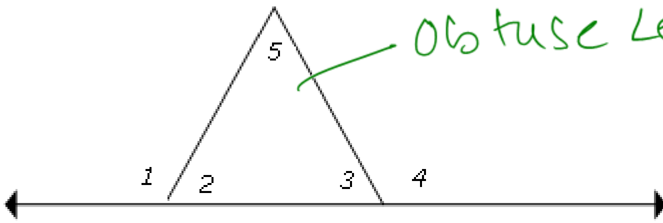


$m\angle 1 = m\angle BAC + m\angle ABC$

$m\angle 2 = m\angle BCA + m\angle ABC$

$m\angle 3 = m\angle ACB + m\angle BAC$

Example 3: Given: $m\angle 1 = 8(x + 2)$
$m\angle 3 = 5x - 3$
$m\angle 5 = 5(x + 1) - 2$
Find *x* and measures of angles 1, 2, 3, 4 and 5.

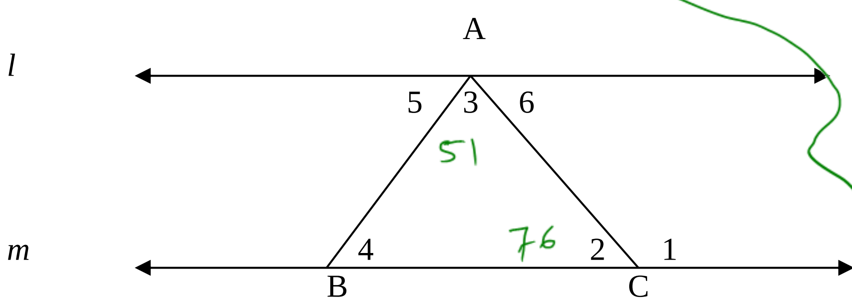


Obtuse $\angle$ed Δ

$m\angle 1 = m\angle 5 + m\angle 3$
$8(x+2) = 5(x+1) - 2 + 5x - 3$
$8x + 16 = 5x + 5 - 2 + 5x - 3$
$8x + 16 = 10x$
$16 = 2x$
$8 = x$

$m\angle 1 = 8(8+2) = 80^\circ$
$m\angle 3 = 5(8) - 3 = 37^\circ$
$m\angle 5 = 5(8+1) - 2 = 43^\circ$
$m\angle 1 + m\angle 2 = 180$
$m\angle 2 = 180 - 80 = 100^\circ$
$m\angle 3 + m\angle 4 = 180$
$m\angle 4 = 180 - 37 = 143^\circ$
$[m\angle 4 = m\angle 2 + m\angle 5]$

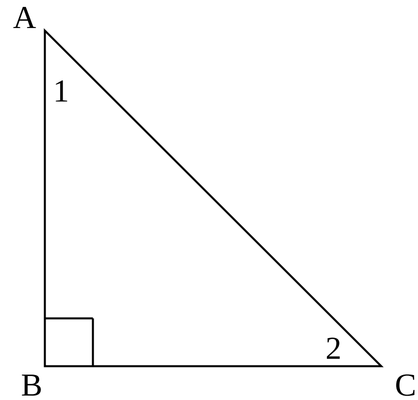
Example 4: Given ΔABC and $l \parallel m$.



Given: $m\angle 3 = 51°$, and $m\angle 2 = 76°$, find $m\angle 1$, $m\angle 4$, $m\angle 5$, and $m\angle 6$,

$m\angle 2 = m\angle 6$
[Int Alt $\angle$s]
$\therefore m\angle 6 = 76$
$m\angle 2 + m\angle 1 = 180$
$m\angle 1 = 180 - 76 = 104^\circ$
$m\angle 3 + m\angle 4 + m\angle 2 = 180$
$m\angle 4 = 180 - 76 - 51 = 53^\circ$
$m\angle 4 = m\angle 5$
$\therefore m\angle 5 = 53^\circ$

Example 5: In the figure provided, find the following:



a. Find $m\angle 1 + m\angle 2$

$m\angle 1 + \angle 2 + \angle B = 180$

$m\angle 1 + m\angle 2 = 90^\circ$ [as $m\angle B = 90^\circ$]

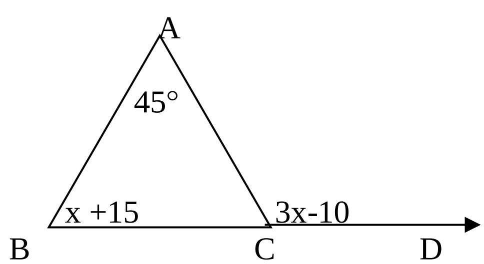
b. Find x if the $m\angle 1 = 4x + 7$ and $m\angle 2 = 2x + 3$

$4x + 7 + 2x + 3 = 90$

$6x + 10 = 90$

$6x = 80 \Rightarrow x = \frac{80}{6} = 13.33$

Example 6; Given ΔABC



Solve for x and give the measure of $\angle ABC$ and $\angle ACD$.

$3x - 10 = 45 + x + 15$

$3x - 10 = 60 + x$

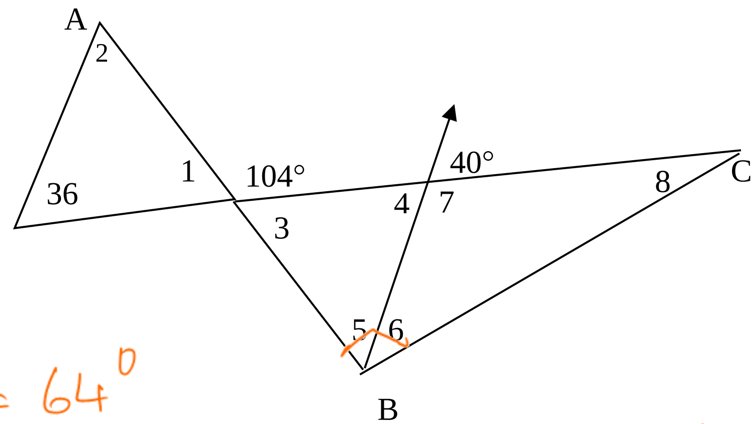
$2x = 70$

$x = 35$

$m\angle ABC = 35 + 15$
$= 50^\circ$

$\therefore m\angle ACD = 3(35) - 10$
$= 95^\circ$

Example 7: If AB is perpendicular to BC, find the measure of each angle in the figure below.



$m\angle 1$ = 180 − 104 = 76°

$m\angle 2$ = 104 − 36 = 68°

$m\angle 3$ = 76° (∠1 & ∠3 VA)

$m\angle 4$ = 40° (VA)

$m\angle 5$ = 180 − 76 − 40 = 64°

$m\angle 6$ = 90 − 64 = 26°

$m\angle 7$ = 180 − 40 = 140°

$m\angle 8$ = 90 − m∠3

= 90 − 76 = 14°

[m∠7 + m∠6 + m∠8 = 180°]

m∠3 + m∠4 + m∠5 = 180

m∠5 = 180 − m∠3 − m∠4

m∠5 + m∠6 = 90

m∠6 = 90 − m∠5

MORE?!?!? Try these: p. 93 #'s 16, 19, 28