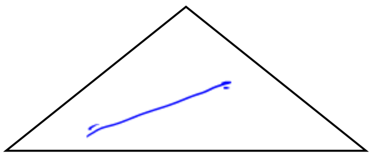
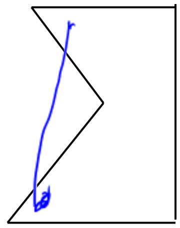
Convex Polygons

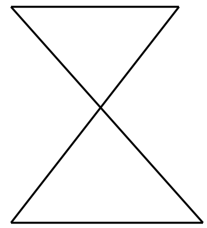
Definition: A polygon is closed plane figure whose sides are line segments that intersect only endpoints.



Convex



Concave

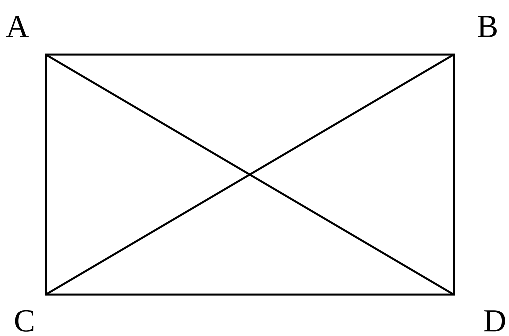


Not a Polygon

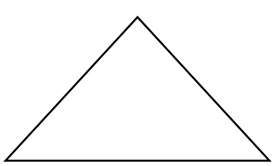
Special names for polygons with fixed numbers of sides *[fill in the polygon column with the names]*

Number of sides	Polygon
3	Triangle
4	Quadrilateral
5	pentagon
6	Hexagon
7	Heptagon
8	octagon
9	Nonagon
10	Decagon
21	21-gon

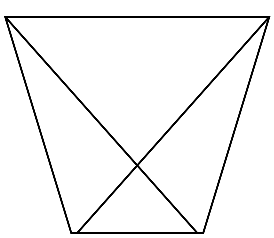
Diagonal is a line segment that join non-adjacent vertices.



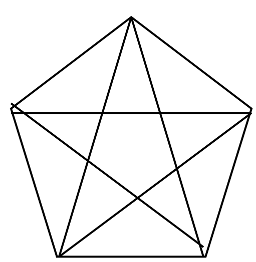
Page 100: Number of diagonals



0 diagonals
triangle



2 diagonals
quadrilateral



5 diagonals
pentagon

Theorem 2.5.1: The total number of diagonals D in a polygon of n sides is given by the formula $D = \frac{n(n-3)}{2}$

Example 1: Given the number of sides of a polygon find the number of diagonals.

a. Triangle

$n=3$

$$D = \frac{3(3-3)}{2} = \frac{3(0)}{2} = \frac{0}{2} = 0$$

b. 11 sided polygon

$n=11$

$$D = \frac{11(11-3)}{2} = \frac{11\cdot \cancel{8}^{4}}{\cancel{2}} = 44$$

Theorem 2.5.2: The sum S of the measures of the interior angles of a polygon with n sides is given by $S = (n-2) \bullet 180°$. Note that n >2 for any polygon.

Example 2: Find the sum of the interior angles of the given polygon.

a. Triangle

$n=3$

$$S = (3-2)180 = 1\cdot 180 = 180^\circ$$

b. 11 sided polygon.

$n=11$

$$S = (11-2)180 = 9(180) = 1620^\circ$$

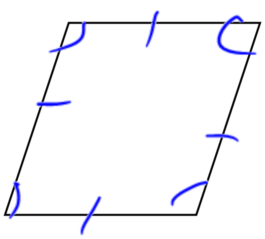
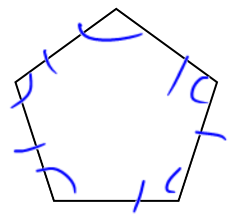
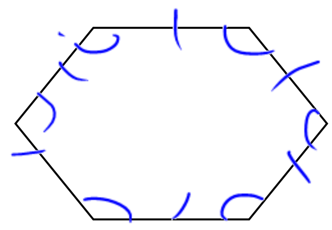
Example 3: Find the number sides a polygon has given the sum of the interior angles.

S = 1980

$$S = (n-2)180$$
$$1980 = (n-2)180$$
$$\frac{1980}{180} = n-2$$
$$11 = n-2$$
$$13 = n$$

Regular Polygons

Definition: A regular polygon is a polygon that is both equilateral and equiangular.

Corollary 2.4.3: the measure I of each interior angle of a regular polygon or equiangular polygon of n sides

$$I = \frac{(n-2)\bullet 180^\circ}{n}$$

Example 4: Find the measure of each of the interior angle of a regular hexagon.

$n = 6$

$$I = \frac{(n-2)\cdot 180}{n} = \frac{(6-2)\cdot 180}{6}$$

$$= \frac{4\cdot \cancel{180}\,30}{\cancel{6}} = 120^\circ$$

Example 5: Each interior angle of a regular polygon is 150°. Find the number of sides.

$$150 = \frac{(n-2)\cdot 180}{n}$$

$$150n = (n-2)\cdot 180$$

$\Rightarrow 150n = 180n - 360$

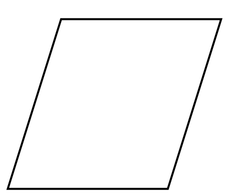
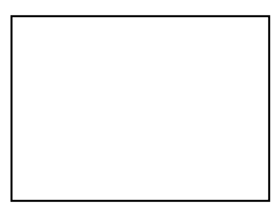
$360 = 180n - 150n$

$360 = 30n \Rightarrow \frac{360}{30} = n$

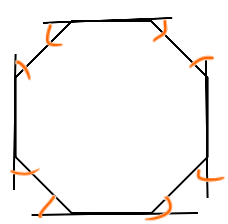
$\Rightarrow 12 = n$

Corollary 2.5.4: The sum of the four interior angles of a quadrilateral is 360°.

$$S = (n-2)\bullet 180^\circ \text{ or } S = (4-2)\bullet 180^\circ = 360^\circ$$

Corollary 2.5.5: The sum of the measured of the exterior angles of a polygon is 360°.



Corollary 2.5.6: The measure E of each exterior angle of a regular polygon of n sides is

$$E = \frac{360°}{n}$$

Example 6: Find the number of sides in a regular polygon whose exterior angles each measure $22.5°$.

$$E = \frac{360}{n} \Rightarrow 22.5 = \frac{360}{n}$$

$$\Rightarrow n = \frac{360}{22.5} = 16$$

Example 7: Find the measure of each interior angle of a stop sign.

$$I = \frac{(n-2)\cdot 180}{n}$$

$$= \frac{(8-2)\cdot 180}{8}$$

$$= \frac{\cancel{6}^{3}\cdot \cancel{180}^{45}}{\cancel{8}_{\cancel{4}}} = 135°$$

$$E = \frac{360}{n} = \frac{360}{8} = 45°$$

$$I = 180 - E = 180 - 45 = 135°$$

Example 8: Find the number of sides that a regular polygon has if the measure of each interior angle is $144°$.

$$I = \frac{(n-2)\cdot 180}{n}$$

$$144 = \frac{(n-2)\,180}{n}$$

$$144n = (n-2)\,180$$

$$144n = 180n - 360$$

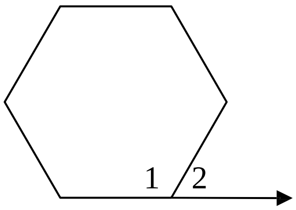
$$\Rightarrow 360 = 180n - 144n$$

$$360 = 36n$$

$$\frac{360}{36} = n$$

$$10 = n$$

Observation: An interior of a polygon angle and an adjacent exterior angle are supplementary.



$m\angle 1 + m\angle 2 = 180$

Example 9: If an interior angle of a regular polygon measures 165°, find

a) the measure of an exterior angle

$m(\text{Exterior}) = 180 - 165 = 15^\circ$

b) the number of sides

$$E = \frac{360}{n}$$

$$15 = \frac{360}{n}$$

$$n = \frac{360}{15} = 24$$

OR

$$I = \frac{(n-2)180}{n}$$

$$165 = \frac{(n-2)180}{n}$$

$$165n = 180n - 360$$

$$360 = 180n - 165n$$

$$360 = 15n$$

$$\frac{360}{15} = n$$

$$24 = n$$

TRY THESE: p 101 #'s 14, 28, 29

Suppos $D = 44$

$$ax^2 + bx + c = 0$$

$$D = \frac{n(n-3)}{2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$44 = \frac{n(n-3)}{2}$$

$a = 1$
$b = -3$
$c = -88$

$$88 = n(n-3)$$
$$88 = n^2 - 3n$$
$$0 = n^2 - 3n - 88$$
$$n^2 - 3n - 88 = 0$$

$a \cdot c = 88$

$2 \cdot 44$
$4 \cdot 22$
$11 \cdot 8$

$$n^2 - 11n + 8n - 88 = 0$$
$$n(n-11) + 8(n-11) = 0$$
$$(n-11)(n+8) = 0$$

$n - 11 = 0$ $\quad$ $n + 8 = 0$

$n = 11$ $\quad$ $n = -8$ X