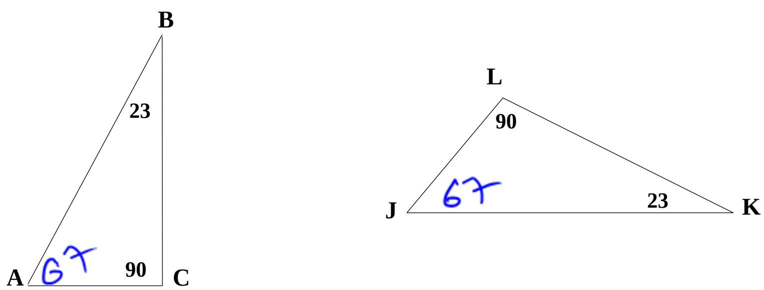
Math 1312 Section 3.1
Congruent Triangles

To have a *correspondence* between two triangles, you must “match up” the angles and sides of one triangle with the angles and sides of the other triangle. Each corresponding angle and side must have the same measure.



The correspondence between the above two triangles can be stated as $\Delta ABC \leftrightarrow \Delta JKL$.

The order in which the letters are written matters since it shows which angles and sides of one triangle match up with the angles and sides of the other triangle:

If $\Delta ABC \leftrightarrow \Delta JKL$, the corresponding angles are:

$$\Delta\, A\, B\, C \;\leftrightarrow\; \Delta\, J\, K\, L$$

AND the corresponding segments are:

$\Delta\, A\, B\, C \leftrightarrow \Delta\, J\, K\, L$ $\Delta\, A\, B\, C \leftrightarrow \Delta\, J\, K\, L$ $\Delta\, A\, B\, C \leftrightarrow \Delta\, J\, K\, L$

$\overline{AB} \leftrightarrow \overline{JK}$ $\overline{BC} \leftrightarrow \overline{KL}$ $\overline{AC} \leftrightarrow \overline{JL}$

The correspondence may be written in more than one way: $\Delta CAB \leftrightarrow \Delta LJK$ is the same as $\Delta ABC \leftrightarrow \Delta JKL$.

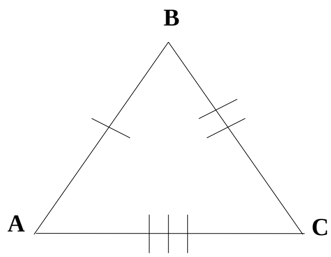
<u>CONGRUENT TRIANGLES</u>

Each triangle has six parts: three sides and three angles. If the six parts of one triangle are congruent to the corresponding six parts of another triangle, then the triangles are *congruent triangles*.

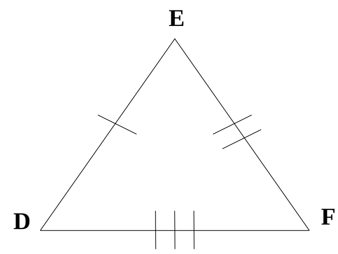
Two triangles are congruent if:

1. All pairs of corresponding angles are congruent.
2. All pairs of corresponding sides are congruent.

Below, ΔABC and ΔDEF are *congruent*, this means that the corresponding parts of each triangle are the same measures. The congruence is written $\Delta ABC \cong \Delta DEF$.



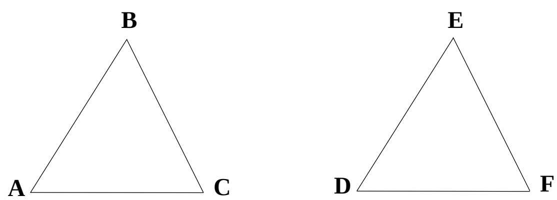
$\angle A \cong \angle D$ $\quad$ $AB \cong DE$
$\angle B \cong \angle E$ $\quad$ $BC \cong EF$
$\angle C \cong \angle F$ $\quad$ $CA \cong FD$



<u>Definition of Congruent Triangles (CPCTC)</u> - two triangles are congruent if and only if their corresponding parts are congruent. (CPCTC - corresponding parts of congruent triangles are congruent)

<u>Example 1:</u>

If $\Delta ABC \cong \Delta DEF$, name the corresponding congruent angles and sides:

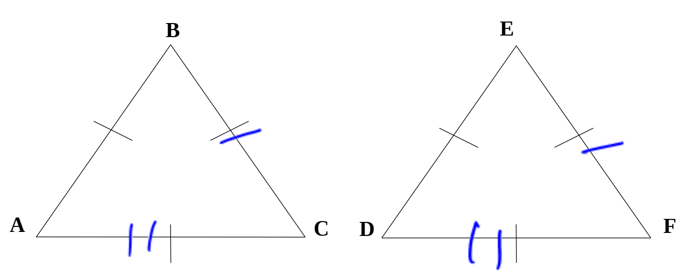


Congruent Angles - $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Congruent Sides - $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$

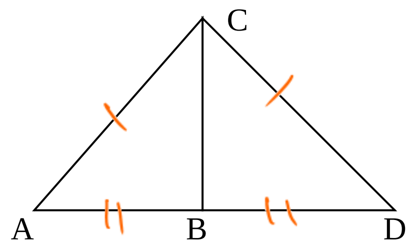
So the real question. Can we have less information and still show two triangles are congruent?

<u>(1) SSS Postulate</u> - (side-side-side) if the three sides of one triangle are congruent to the three sides of a second triangle, then the triangles are congruent.



Since all three sides in ΔABC are congruent to all three sides in ΔDEF, then $\Delta ABC \cong \Delta DEF$

Example 2: Here is an example of SSS. If the lines AC = DC and AB = BD. Show that $\Delta ABC \simeq \Delta DBC$ using SSS.



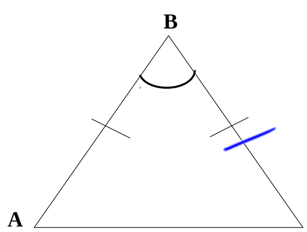
$\overline{BC} \cong \overline{BC}$ (Reflexive)

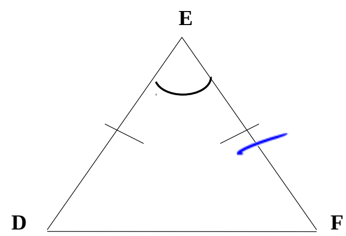
$\therefore ABC \cong \Delta DBC$

SSS

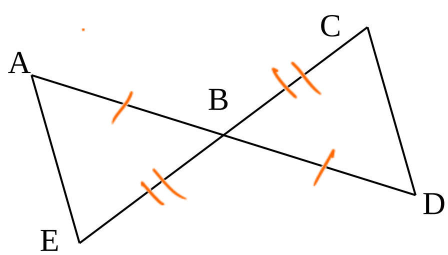
(2) SAS Postulate - if two sides and the "included" angle of one triangle are congruent to two sides and the "included" angle of another triangle, then the triangles are congruent.

In the above definition, the "included" angle is the angle that is formed by the intersection of two sides of a triangle. For example, below ∠B and ∠E are the "included" angles.





Example 3: Show that $\Delta ABE \simeq \Delta DBC$ using SAS.



B is the midpoint of both $\overline{\mathbf{AD}}$ and $\overline{\mathbf{EC}}$

$\overline{AB} \cong \overline{BD}$ } Def of mid pt
$\overline{EB} \cong \overline{BC}$

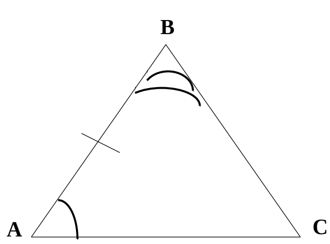
$\angle ABE \cong \angle CBD$ (VA)

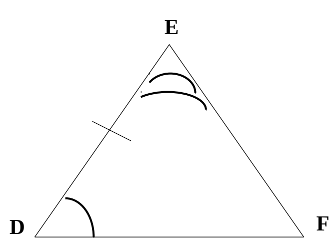
$\therefore \Delta ABE \cong \Delta DBC$

SAS

(3) ASA Postulate - if two angles and the "included" side of one triangle are congruent to two angles and the "included" side of another triangle, the triangles are congruent.

In the above definition, the "included" side is the segment that connects two angles. For example, AB and DE are the "included" sides .

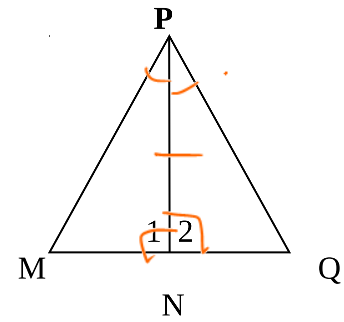




Example 4: Given the following information:

$\overline{PN} \perp \overline{MQ}$; $\angle MPN \cong \angle NPQ$

$\Delta PNM \cong \Delta PNQ$ by ASA



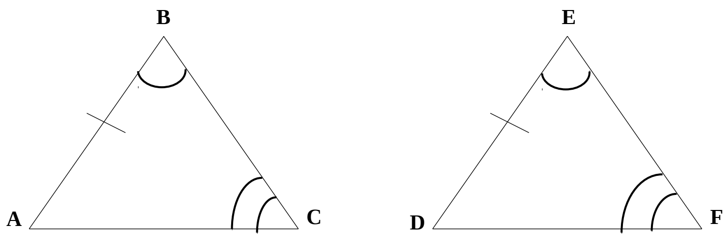
$\angle MPN \cong \angle NPQ$ (Given)

$\angle PNM \cong \angle PNQ$ (as $\overline{PN} \perp \overline{MQ}$)

$\overline{PN} \cong \overline{PN}$ (Reflexive)

$\therefore \Delta PNM \cong \Delta PNQ$ ASA

(4) AAS Theorem - if two angles and a "non-included" side of one triangle are congruent to the corresponding two angles and side of a second triangle, the two triangles are congruent.

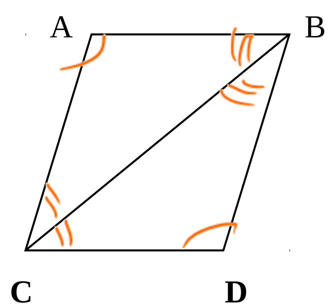


Example 5: Given the following information:

$\angle A \cong \angle D$

$\overline{BC}$ bisects both $\angle ACD$ and $\angle ABD$

How can we show congruence using AAS?



$\angle A \cong \angle D$ Given

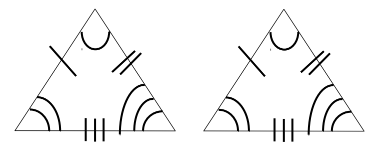
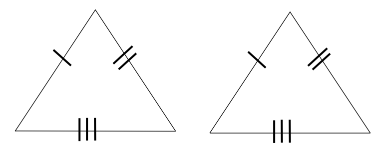
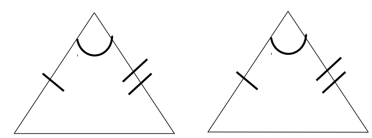
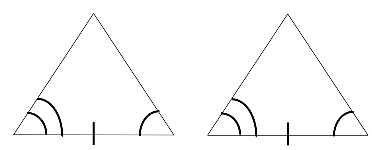
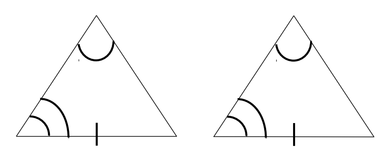
$\angle ACB \cong \angle BCD$ (as $\overline{BC}$ bisects)

$\overline{BC} \cong \overline{BC}$ (Reflexive)

$\therefore \Delta ABC \cong \Delta DBC$ AAS

Note: You can not use AAA and SSA because they are not valid for proving triangles are congruent. A triangle can have equal angles can have the same shape but the triangles are not necessarily congruent.

SUMMARY OF METHODS:

METHOD	QUALIFICATIONS	PICTURE
Def. of $\cong \Delta$	All six parts of one triangle must be congruent with all six parts of the other triangle.	
SSS	The three sides of one triangle must be congruent to the three sides of the other triangle.	
SAS	Two sides and the included angle of one triangle must be congruent to two sides and the included angle of the other triangle.	
ASA	Two angles and the included side of one triangle must be congruent to two angles and the included side of the other triangle.	
AAS	Two angles and a non-included side of one triangle must be congruent to the corresponding two angles and non-included side of the other triangle.	

Try these: 3.1 #'s 5, 9 – 12, 13, 19, 21