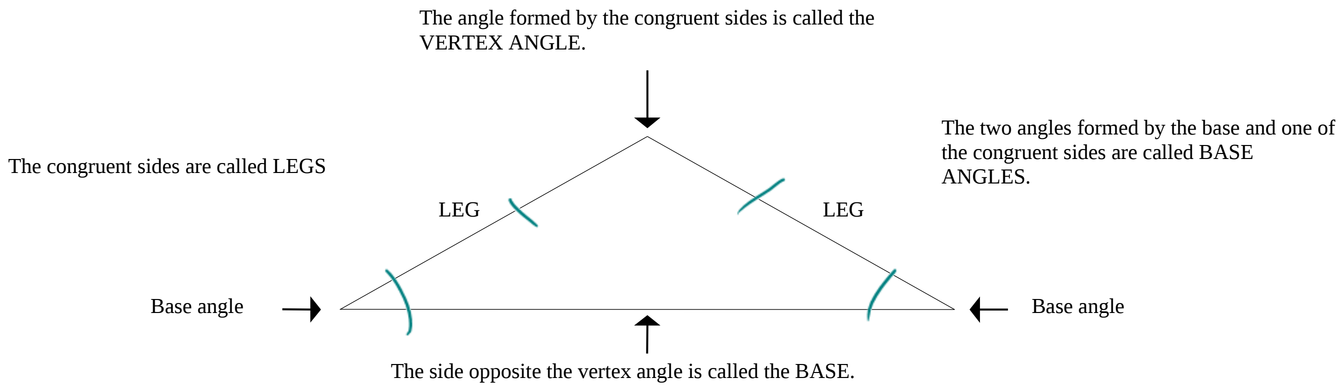
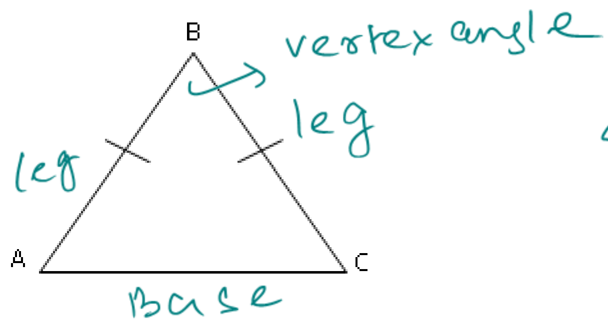
Class Notes
Section 3.3
Analyzing Isosceles Triangles

In an **isosceles triangle**, the two sides that are of equal length are called the **legs** and the third side is called the **base**. The point at which the legs meet is the **vertex** and the angle there is the **vertex angle**. The two angles that include the base are called the **base angles**.



Name the parts of this isosceles triangle:



∠A & ∠C as base angles

Other important triangle parts:

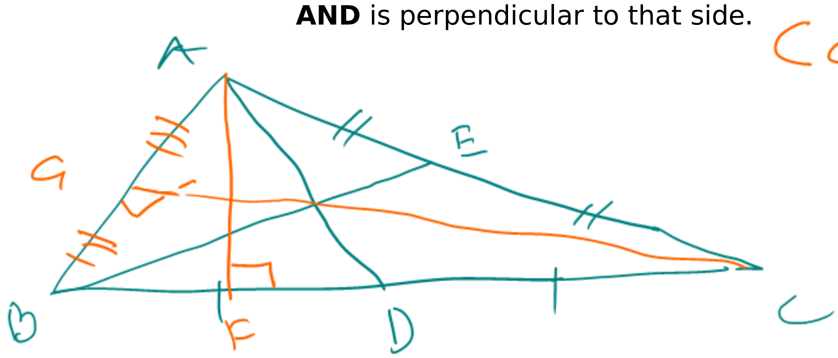
Definitions:

median - a segment that starts from an angle and goes to the midpoint of the opposite side. AD, BE

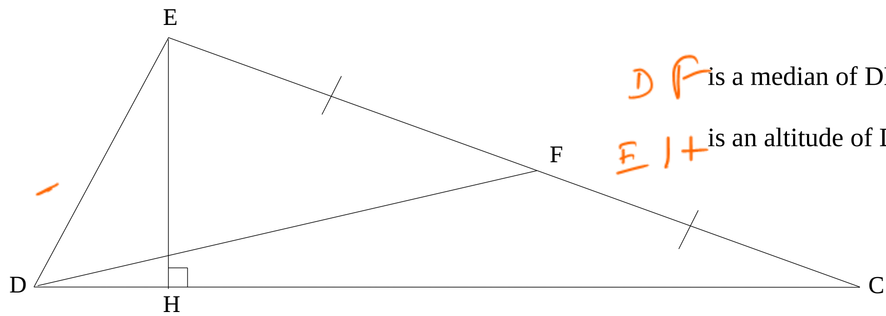
altitude - a segment that starts from an angle and is perpendicular to the opposite side. AF, CG

angle bisector - of a triangle…is a segment that bisects an angle and goes to the opposite side.

perpendicular bisector - a segment that passes through the midpoint of a side **AND** is perpendicular to that side. CG (If AG = GB)



Examples:

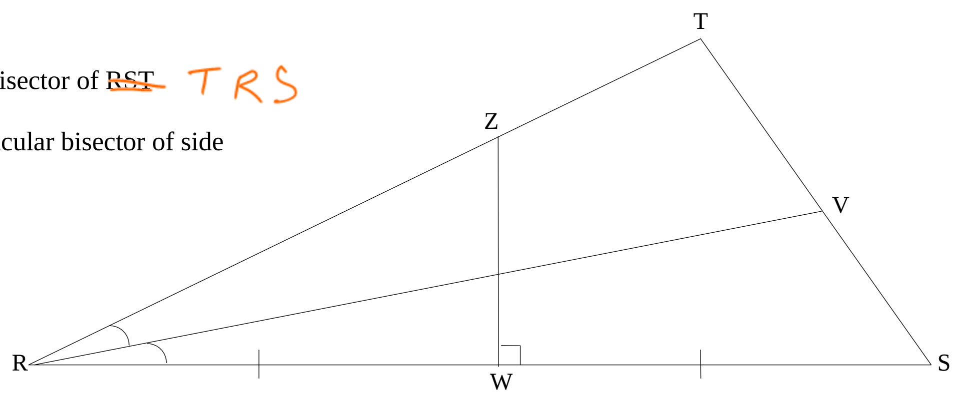


DF is a median of DEC

EH is an altitude of DEC

RV is an angle bisector of ~~RST~~ TRS

ZW is a perpendicular bisector of side



Thm – Corresponding altitudes of congruent triangles are congruent.

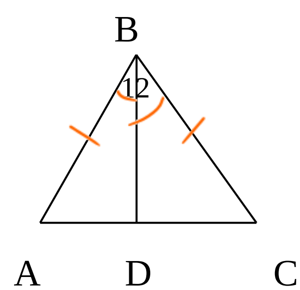
Thm – The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.

Proof –

Given : Isosceles ΔABC, with $\overline{AB} \simeq \overline{BC}$

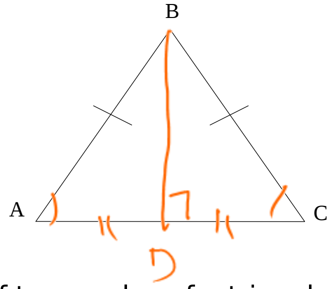
$\overline{BD}$ bisects $\angle ABC$

Prove: $\Delta ABD \simeq \Delta CBD$



statement	Reason
1) ΔABC is isos. $\overline{AB} \cong \overline{BC}$, $\overline{BD}$ bisects $\angle ABC$	1) Given
2) $\angle 1 \cong \angle 2$	2) Def of angle bisector
3) $\overline{BD} \cong \overline{BD}$	3) Reflexive
4) $\Delta ABD \cong \Delta CBD$	4) SAS

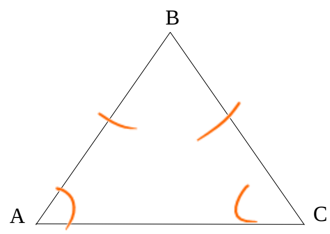
<u>Isosceles Triangle Theorem</u> - if two sides of a triangle are congruent, then the angles opposite those sides are congruent. . . . . .



If $\overline{AB} \cong \overline{BC}$

Then ~~C A~~ $\angle C \cong \angle A$ (CPCTC)

<u>AND (converse)</u> if two angles of a triangle are congruent, then the sides opposite those angles are congruent.

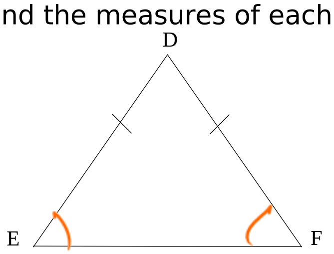


If ~~A C~~ $\angle A \cong \angle C$

Then $\overline{BA} \cong \overline{BC}$

Example 1:

ΔDEF is isosceles. $\angle D$ is the vertex angle. The $m\angle E = 2x + 40$ and the $m\angle F = 3x + 22$. Find the measures of each angle.



$m\angle E = m\angle F$

$2x + 40 = 3x + 22$

$18 = x$

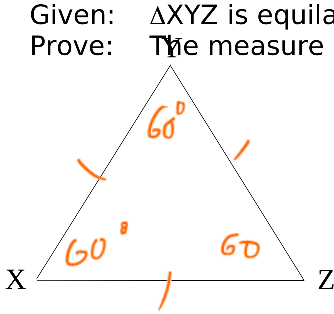
$m\angle E = 2(18) + 40 = 76^\circ$

$m\angle F = 76^\circ$

$m\angle D = 180 - 76 - 76$
$= 28^\circ$

Example 2:

Given: ΔXYZ is equilateral
Prove: The measure of each angle of ΔXYZ is 60°



1) First, we have three congruent sides (given); so, their opposite angles are congruent.

2) Now we have three congruent angles. So, the triangle is equiangular.

3) So, we have an equilateral **AND** equiangular triangle.

5) How can we prove the three angles are each 60°? (hint: what does the sum of the angles of a triangle add up to?)

<u>SO…….</u>

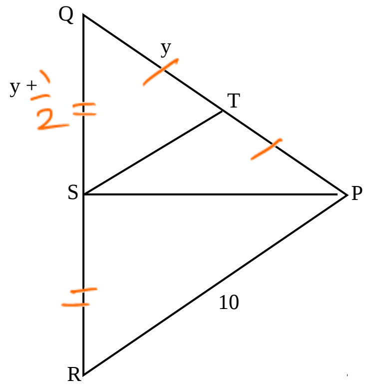
- a triangle is equilateral **if and only if** it is equiangular.
- each angle of an equilateral triangle measures 60°.

Defn – The perimeter of a triangle is the sum of the lengths of all of its sides.

Study the chart on page 145.

Example 3:

a. In the figure below, $\overline{PQ} \cong \overline{PR}$, and $\overline{PS}$ and $\overline{ST}$ are medians. Find QT and QR.



$PQ = PR \therefore PQ = 10$

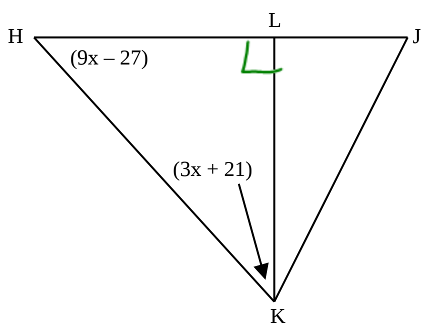
$QT = \frac{1}{2} PQ = \frac{1}{2}(10) = 5$

$y = QT \therefore y = 5$

$QS = y + \frac{1}{2} = 5 + \frac{1}{2} = 5.5$

$QR = 2(QS) = 2(5.5) = 11$

b. $\overline{KL}$ is an altitude of ΔHJK. Find "x".



$m\angle LHK + m\angle LKH = 90^\circ$

$9x - 27 + 3x + 21 = 90$

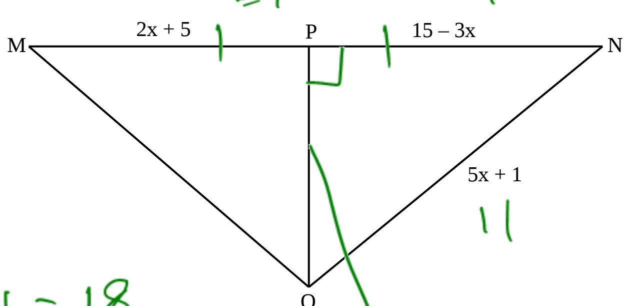
$12x - 6 = 90$

$12x = 96$

$x = 8$

c. $\overline{PO}$ is the perpendicular bisector of $\overline{MN}$. Find "x".

$2(2)+5 = 9$ 9



$MP = PN$

$2x + 5 = 15 - 3x$

$5x = 10$

$x = 2$

$MN = 18$

$\sqrt{11^2 - 9^2}$

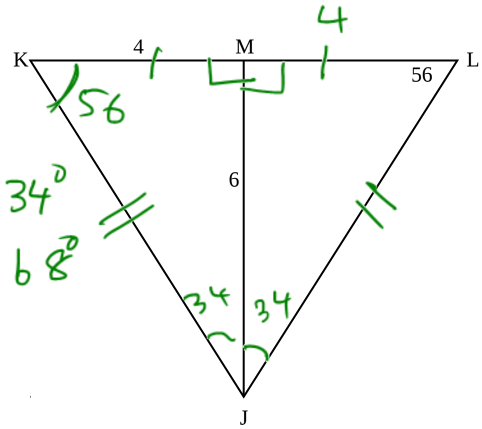
$= \sqrt{121 - 81}$

$= \sqrt{40}$

$= 2\sqrt{10}$

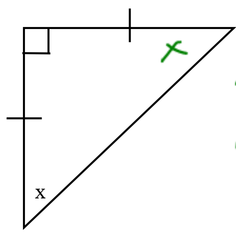
d. In $\triangle JKL$, $\overline{JK} \cong \overline{JL}$, and $\overline{JM}$ is both a median, and altitude, and an angle bisector. Find the following.

1. $m\angle KMJ$ 90
2. KL 8
3. $m\angle KJM$ $90 - 56 = 34^\circ$
4. $m\angle KJL$ $34 \times 2 = 68^\circ$
5. $m\angle K$ 56°



Example 4:

a. x = 45°

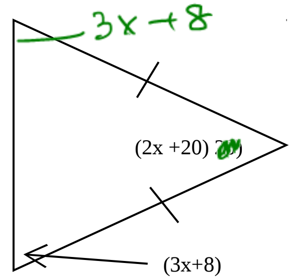


$x + x = 90$

$2x = 90$

$x = 45$

b. x = 18



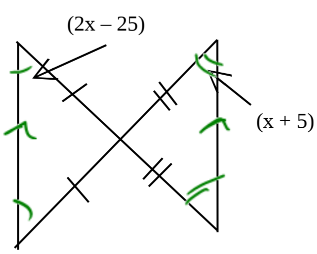
$3x + 8 + (2x + 20) + 3x + 8 = 180$

$8x + 36 = 180$

$8x = 144$

$x = 18$

c. x = 30



$2x - 25 = x + 5$

$x = 30$

d. Use the figure below to find the angle measures if $m\angle 1 = 30$.

$m\angle 2$ = __________

$m\angle 3$ = __________

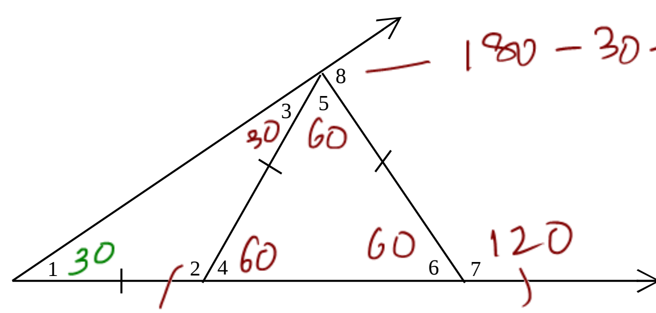
$m\angle 4$ = __________

$m\angle 5$ = __________

$m\angle 6$ = __________

$m\angle 7$ = __________

$m\angle 8$ = __________



$180 - 30 - 60 = 90$

$180 - 30 - 30 = 120$

$180 - 60$