Inequalities in a Triangle

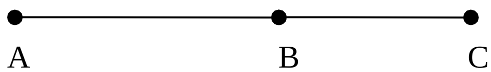
Definition: Let a and b be real numbers a > b if and only if there is a positive number p for which a = b + p

Example 1: 7 > 2 and 5 is a positive number so 7 = 2 + 5. 7 > 2 and 7 > 5

a b a b p

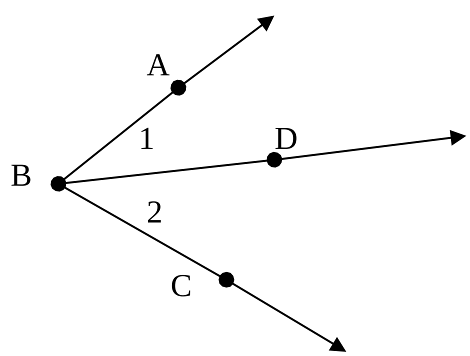
Lemma 3.5.1: If B is between A and C on $\overline{AC}$, then AC > AB and AC > BC. (The measure of a line segment is greater than the measure of any its parts.)

Example 1: According to the segment addition property AC = AB + BC

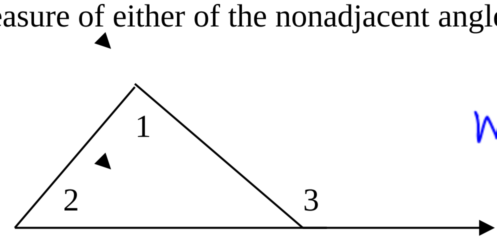


Lemma 3.5.2: If $\overrightarrow{BD}$ separates $\angle ABC$ into two parts ($\angle 1$ and $\angle 2$),then the measure of $m\angle ABC$ > $\angle 1$ and $m\angle ABC$ > $\angle 2$. (The measure of an angle is greater than the measure of any its parts.)

Example 2: According to the angle addition postulate $\angle ABC = \angle 1 + \angle 2$



Lemma 3.5.3: If $\angle 3$ is an exterior angle of a triangle and $\angle 1$ and $\angle 2$ are non adjacent interior angles, then, $m\angle 3$ > $m\angle 1$ and $m\angle 3$ > $m\angle 2$. The measure of an exterior angle is greater than the measure of either of the nonadjacent angles.)



$m\angle 3 = m\angle 1 + m\angle 2$

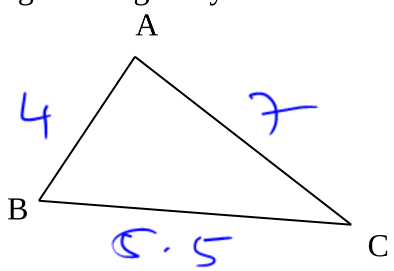
Lemma 3.5.4: In ΔABC, if $\angle C$ is aright angle or an obtuse angle , then the measure of $m\angle C > m\angle B$ and $m\angle C > m\angle A$.

Proof:
In ΔABC, $m\angle A + m\angle B + m\angle C = 180°$. With the $m\angle C \geq 90°$, it follows the $m\angle A + m\angle B \leq 90°$, and each angle ($m\angle A$ and $m\angle B$) must be acute. Thus $m\angle C > m\angle B$ and $m\angle C > m\angle A$.

Lemma 3.5.5: (Addition Property of Inequality) If a > b and c > d, then a + c > b + d.

Theorem 3.5.6: If one side of a triangle is longer than a second side then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.

Example 3: Given the ΔABC with sides of the following lengths AB =4, AC = 7 and BC = 5.5. Arrange the angles by size.

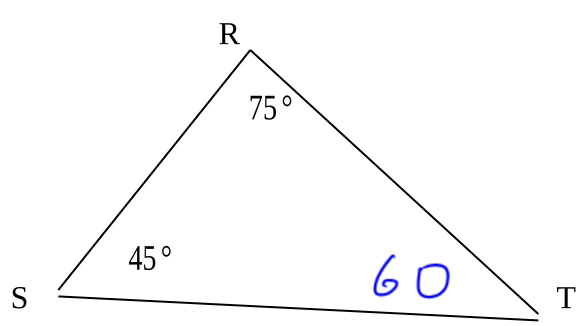


$\overline{AC} > \overline{BC} > \overline{AB}$

$m\angle B > m\angle A > m\angle C$

Theorem 3.5.7: If the measure of one angle of a triangle is greater than the measure of a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Example 4: Given ΔRST with angles that have the following measures. Arrange the sides in order of size.

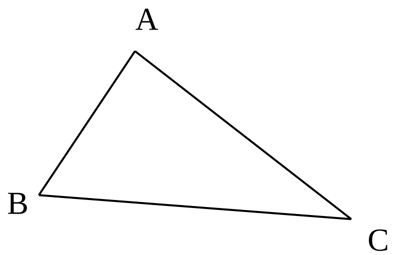


$m\angle R > m\angle T > m\angle S$

$\overline{ST} > \overline{RS} > \overline{RT}$

Theorem 3.5.10: (Triangle Inequality) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Given: ΔABC



BA + CA > BC

Theorem 3.5.10: (Triangle Inequality) The length of any side of a triangle must be between the sum and difference of the other two sides.

Example 5: Which of the following sets of numbers cannot represent the sides of a triangle?

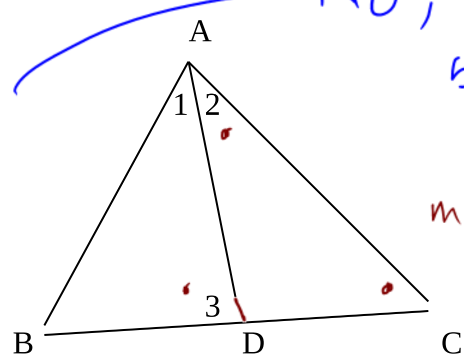
a. 7, 7, 3

$7+7=14>3$ ✓
$7+3=10>7$ ✓
$7+3=10>7$ ✓
Yes

b. 4, 5, 1

$4+5=9>1$ ✓
$4+1=5 \not> 5$ X
No, a Δ cannot be formed

Example 6:



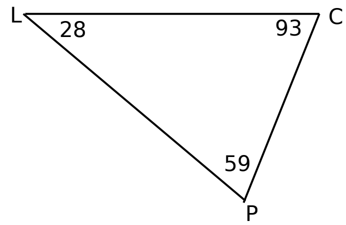
$m\angle 3 = m\angle 2 + m\angle C$

Given: ~~$\overline{BD}$~~ $\overline{AD}$ bisects ~~$\angle ABC$~~ $\angle BAC$
Prove: AB > ~~AD~~ $\overline{BD}$

Statements	Reasons
1. ~~$\overline{BD}$~~ $\overline{AD}$ bisects ~~$\angle ABC$~~ $\angle BAC$	1. Given
2. $m\angle 1 = m\angle 2$	2. Def of $\angle$ bisector
3. $m\angle 3 > m\angle 2$	3 $\angle 3$ is the ext $\angle$ to ΔADC
4. $m\angle 3 > m\angle 1$	4. Substitution
5. $\overline{AB}$ > ~~AD~~ $\overline{BD}$	5. Opp side of Larger $\angle$ (3.5.7)

Example 7:

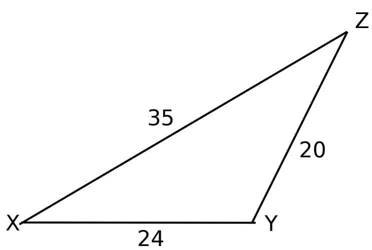
A. Arrange sides from smallest to largest



$m\angle L < m\angle P < m\angle C$

$\overline{CP} < \overline{CL} < \overline{LP}$

B. Arrange angles from smallest to largest.



$\overline{ZY} < \overline{XY} < \overline{XZ}$

$m\angle X < m\angle Z < m\angle Y$

Example8:

List the sides of ΔABC in order from longest to shortest.

$m\angle A = 4x + 20$
$m\angle B = 2x + 10$
$m\angle C = 4x - 20$

$m\angle A + m\angle B + m\angle C = 180$

$4x + 20 + 2x + 10 + 4x - 20 = 180$

$10x + 10 = 180$

$10x = 170$

$x = 17$

$m\angle A = 4(17) + 20 = 88^\circ$

$m\angle B = 2(17) + 10 = 44^\circ$

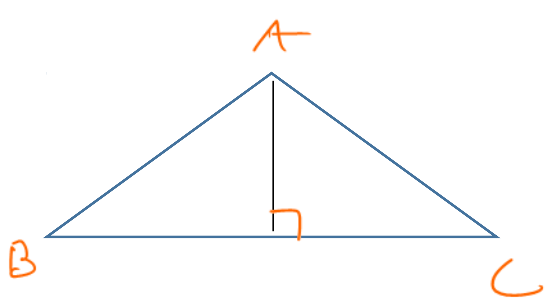
$m\angle C = 4(17) - 20 = 48^\circ$

$m\angle A > m\angle C > m\angle B$

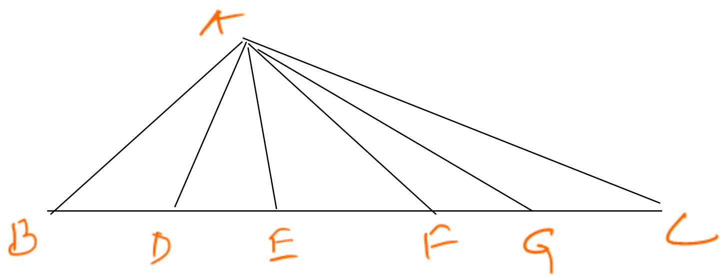
$\overline{BC} > \overline{AB} > \overline{AC}$

Determined, Underdetermine and Overdetermined:

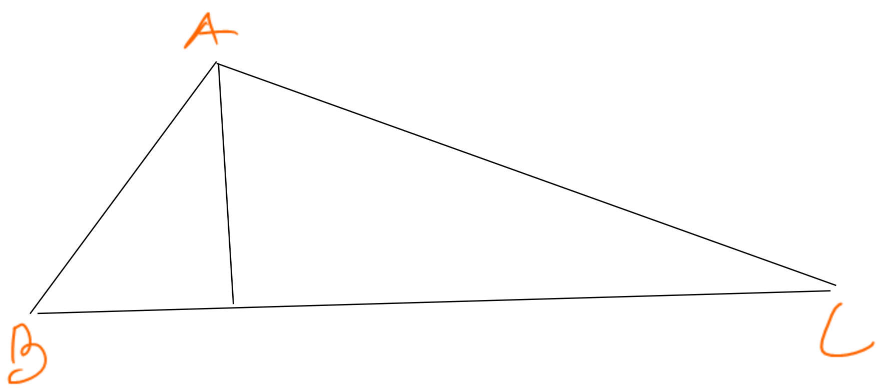
Determined: Draw a line segment from A perpendicular to $\overline{BC}$ so that the terminal point is on $\overline{BC}$. (Determined because the line from A perpendicular to $\overline{BC}$ is unique.



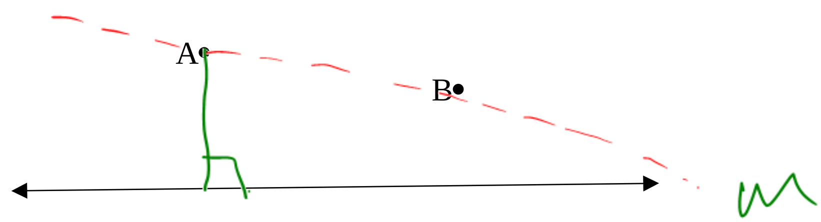
Underdetermine: Draw a line segment from A to $\overline{BC}$ so that the terminal point is on $\overline{BC}$. (Undetermined because many line segments are possible.



Overdetermined: Draw a line segment from A to $\overline{BC}$ so that it bisects $\overline{BC}$. (Overdetermined because the line segment from A drawn perpendicular to $\overline{BC}$ will not contain the midpoint M of $\overline{BC}$.



Example 9:



a. Draw a line segment from *A* perpendicular to *m*. Describe the segment as determined, underdetermined, or overdetermined.

b. Draw a segment $\overline{AB}$ parallel to line *m*. Describe the segment as determined, underdetermined, or overdetermined.