4.2 The Parallelogram and Kite

Theorems 4.2.1-4.2.3 Have the form of "If ______ then this quadrilateral is a parallelogram."

We will find that quadrilaterals having certain characteristics must be parallelogram.

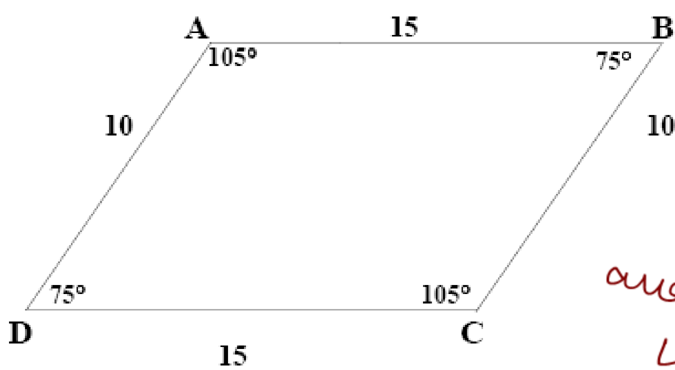
Theorem 4.2.1: If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

Theorem 4.2.2: If both pairs of a opposite sides of a quadrilateral are congruent then it is a parallelogram.

Theorem 4.2.3: If the diagonal of a quadrilateral bisect each other then the quadrilateral is a parallelogram.

Example 1:

Is ABCD a quadrilateral ? Why? Is ABCD a parallelogram? Why?



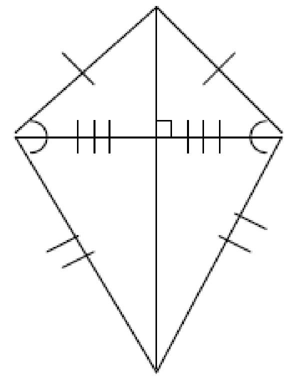
Yes
Opp ∠s are ≅
and opp sides are ≅
4.2.2

Kite! A quad with 2 distinct consecutive side ≅

Definition: A kite is a quadrilateral with two distinct pairs of congruent angles.

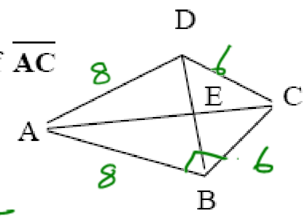
Theorem 4.2.4: In a kite, **one** pair of opposite angles are congruent.

KITES



- Not a parallelogram
- Two pairs of consecutive congruent sides
- The diagonals are perpendicular.
- Exactly one pair of opposite angles are congruent.
- One diagonal is a perpendicular bisector of the other.

Example 2: Given a kite ABCD, $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$:

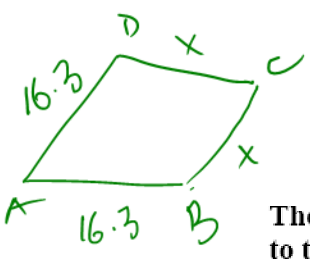


a. If $\angle B = 90°$ and AB =8 and BC 6. Find the length of $\overline{AC}$

$AC^2 = AB^2 + BC^2$

$= 8^2 + 6^2$

$= 64 + 36 = 100 \quad \therefore AC = \sqrt{100} = 10$



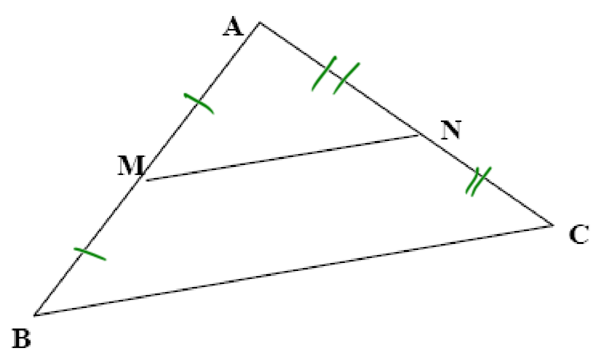
b. If AB = 16.3 and the perimeter of the kite is 54.7, find the lengths of DC, BC and AD.

$16.3 + 16.3 + x + x = 54.7$

$32.6 + 2x = 54.7 \qquad x = 11.05$

$2x = 22.1$

Theorem 4.2.5: The segment that joins the midpoints of the two sides of a triangle is parallel to the third side and has a length equal to on half the length of the third side.

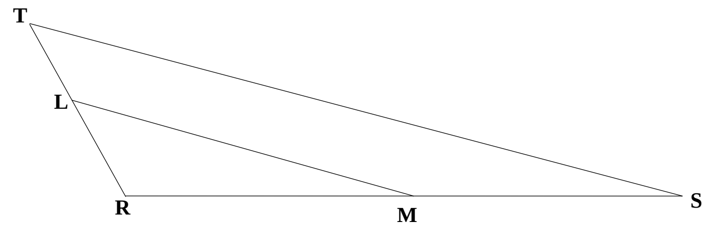


$\overline{MN} \parallel \overline{BC}$

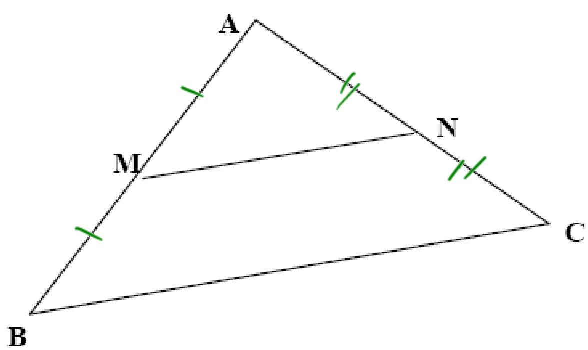
$MN = \frac{1}{2} BC$

<u>Clarification</u>: in ΔTRS "M" is the midpoint of $\overline{RS}$ and "L" is the midpoint $\overline{RT}$.

By the above "rule", ML ll ST and ML = ½ ST. This can also be expressed as 2ML = ST.



Example 3. M and N are the midpoints of the sides $\overline{AB}$ and $\overline{BC}$ of ΔABC



$MN = \frac{1}{2} BC$

$x + 9 = \frac{1}{2}(4x + 6)$

$2(x+9) = 4x + 6$

$2x + 18 = 4x + 6$

$12 = 2x$

$6 = x$

a. If MN = 7.3, find the length of $\overline{BC}$.

$MN = \frac{1}{2} BC \Rightarrow 7.3 = \frac{1}{2} BC$

$\Rightarrow BC = 2(7.3)$

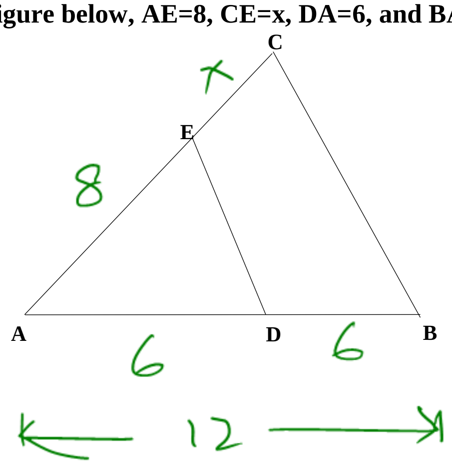
$= 14.6$

b. If BC = 4x + 6 and MN = x + 9, find the length of $\overline{BC}$.

$BC = 4(6) + 6 = 30$ $\quad MN = 15$

<u>Example 4:</u>

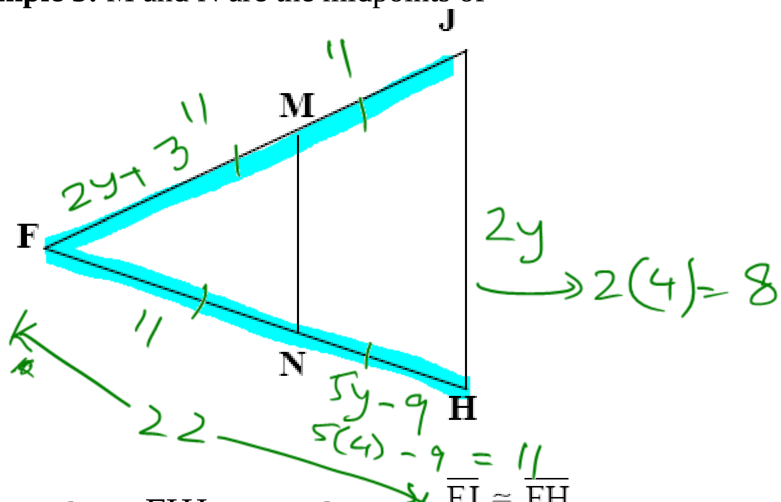
In the figure below, AE=8, CE=x, DA=6, and BA=12. Is ED ll CB?



ED ll CB if E & D are mid pts of AC & AB respectively

So ED ll CB if x = 8

Example 5: M and N are the midpoints of $\overline{FJ}$ and $\overline{FH}$



a. Given that ΔFHJ is isosceles, with $\overline{FJ} \cong \overline{FH}$, FM = 2y +3, NH = 5y−9 and JH = 2y.
Find the perimeter of ΔFHJ .

FM = MJ = FN = NH

FM = NH

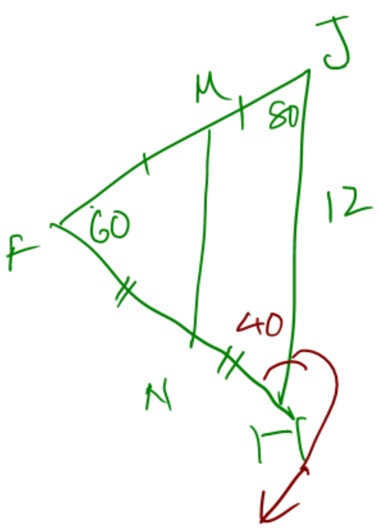
2y+3 = 5y - 9

12 = 3y

4 = y

Perimeter = 22+22+8 = 52

b. Given JH = 12, $m\angle J = 80°$ and $m\angle F = 60°$. Find MN, $m\angle FMN$ and $m\angle FNM$.



$MN = \frac{1}{2} JH$

$= \frac{1}{2}(12) = 6$

$MN \parallel JH$

$\therefore m\angle FMN = m\angle FJH$ (Corres ∠s)

$= 80°$

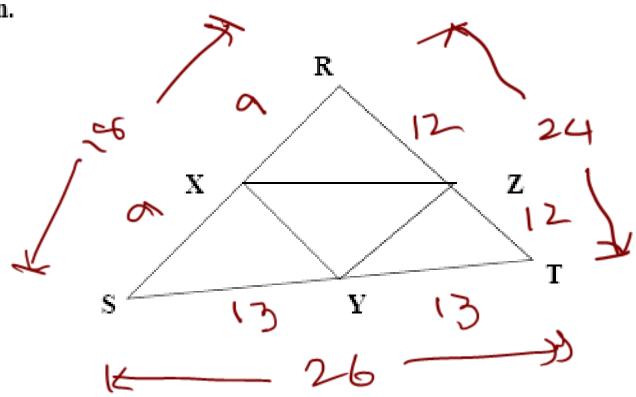
180 - 60 - 80

= 40

$\therefore m\angle FNM = m\angle FHJ$ (Corres ∠s)

$= 40°$

Example 6: Use the following figure for both parts a and b. In ΔRST, X, Y and Z are the midpoints of the sides as shown.



a. If RS = 18 , RT =24, and ST = 26. Find XY, YZ, XZ and the perimeter of ΔXYZ.

$XZ = \frac{1}{2} ST$ $\quad XY = \frac{1}{2} RT$ $\quad YZ = \frac{1}{2} RS$

$= 13$ $\quad = 12$ $\quad = 9$

$\therefore$ Perimeter of $\Delta XYZ = XY + XZ + YZ$

$= 12 + 13 + 9 = 34$

b. If XY = 7.2 , XZ = 6.9, YZ = 5.1. Find RS, RT, ST and perimeter ΔRST.

$RT = 2(7.2) = 14.4$ $\quad ST = 2(6.9) = 13.8$ $\quad SR = 10.2$

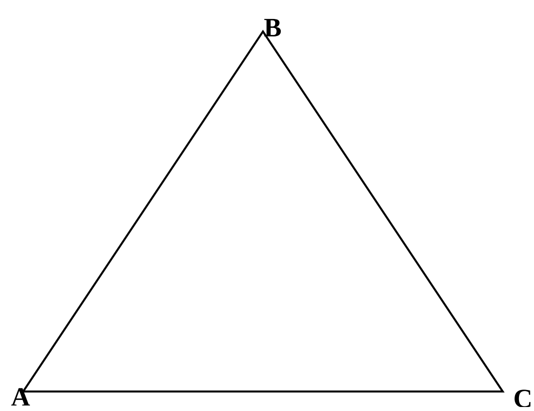
Example 7:

$\therefore$ Perimeter $\Delta RST = 14.4 + 13.8 + 10.2$

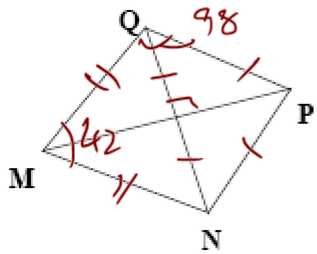
$= 38.4$

In ΔABC, D is the midpoint of AB, E is the midpoint of BC, and F is the midpoint of AC. Find the perimeter of ΔDEF if AB = 24, BC = 32, and AC = 26.

Perimeter of ΔDEF = ___________



Example 8: In kite MNPQ. $\overline{MP}$ is the perpendicular bisector of $\overline{NQ}$. If $m\angle QMN = 42°$ and $m\angle MNP = 98°$, find $m\angle NPQ$.



$\triangle MQP \cong \triangle MNP$ (SSS)

$\overline{MN} \cong \overline{MQ}$ } given
$\overline{NP} \cong \overline{QP}$ }
$\overline{MP} \cong \overline{MP}$ Ref

$\angle MNP \cong \angle MQP$ (CPCTC)

$\therefore m\angle MNP = 98°$

Sum of int $\angle$s in a quad 360

$\therefore m\angle NPQ = 360 - 42 - 98 - 98$
$= 122°$