Rectangle

Definition: A rectangle is a parallelogram that has a right angle.

Corollary 4.3.1: All angles of a rectangle are right angles.

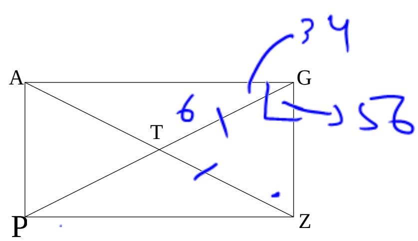
Theorem 4.3.2: The diagonals of a rectangle are congruent.

A rectangle is a parallelogram:

1) Opposite sides are congruent (they equal each other).
2) Opposite angles are congruent (they equal each other).
3) Consecutive angles are supplementary (they add up to 180)
4) Diagonals bisect each other (they cut each other in half)
5) Diagonals are congruent (they equal each other)
6) All four angles are 90.

Example 1:

Given: Rectangle AGPZ



a. $\overline{GT} = 6$. Find $\overline{AZ}$.

$GP = 2(GT) = 2(6) = 12$

$GP = AZ$ (as diag are $\cong$) $\therefore$ $AZ = 12$

b. $m\angle AGP = 34$. Find $m\angle GZA$.

$m\angle ZGP = 90 - 34 = 56^\circ$

$AZ = GP \Rightarrow \frac{1}{2} AZ = \frac{1}{2} GP \Rightarrow TZ = GT$ $\therefore$ $\triangle GTZ$ is isos

$\therefore m\angle GZA = 56$

c. $\overline{AT} = 3x + 13$ and $\overline{TG} = 5x - 21$. Find $\overline{GP}$.

$AT = TG \Rightarrow 3x + 13 = 5x - 21$

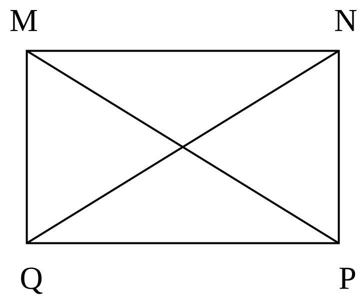
$\Rightarrow 34 = 2x$

$\Rightarrow 17 = x$

$TG = 5(17) - 21 = 64$

$GP = 2(GT) = 2(64) = 128$

Example 2: Given the rectangle



a. If QP = 9 and NP = 6, find NQ and MP.

$$3\,\underline{|117}$$
$$3\,\underline{|39}$$
$$13$$

$NQ^2 = PQ^2 + NP^2$

$= 9^2 + 6^2$

$= 81 + 36$

$\therefore NQ = \sqrt{117}$

$= \sqrt{9 \cdot 13} = \sqrt{9} \cdot \sqrt{13} = 3\sqrt{13}$

NQ = MP (as diag of Rec are $\cong$)

$\therefore MP = 3\sqrt{13}$

b. If MQ = x , MP = 51 and QP = 2 x, find x and the length of QP.

$MP^2 = MQ^2 + QP^2$

$(51)^2 = x^2 + (2x)^2$

$51^2 = x^2 + 4x^2$

$51^2 = 5x^2$

$\frac{51^2}{5} = x^2$

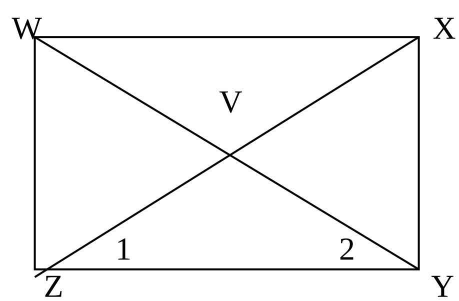
$\sqrt{\frac{51^2}{5}} = x \Rightarrow \frac{51}{\sqrt{5}} = x$

$\Rightarrow \frac{51\sqrt{5}}{\sqrt{5}\sqrt{5}} = x \Rightarrow \frac{51\sqrt{5}}{5} = x$

Example 3:

Given : Rectangle WXYZ with diagonals $\overline{WY}$ and $\overline{XZ}$.

Prove: $m\angle 1 \cong m\angle 2$



Statements	Reasons
1. Rectangle WXYZ with diagonals $\overline{WY}$ and $\overline{XZ}$	1. Given
2. $\overline{WY} \cong \overline{XZ}$	2 The diagonals of a rectangle are $\cong$
3. $\overline{WZ} \cong \overline{XY}$	3. Opposite sides of a rectangle are $\cong$
4. $\overline{ZY} \cong \overline{ZY}$	4. Reflexive
5. $\Delta XZY \cong \Delta WYZ$	5. SSS
6. $m\angle 1 \cong m\angle 2$	6. CPCTC

Example 4: Given: rectangle QRST and Parallelogram QZRC, find length of RZ, ZQ and CS if
RZ = 6x, ZQ = 3x +2y and CS = 14-x

QC = CR (as they half of the diag of the rec)

QC = ZR , QZ = CR (opp sides of a llgram are ≅)

∴ QC = ZR = QZ = CR

QC = CS = CR = CT

$RZ = ZQ$

$6x = 3x + 2y$

$6(2) = 3(2) + 2y$

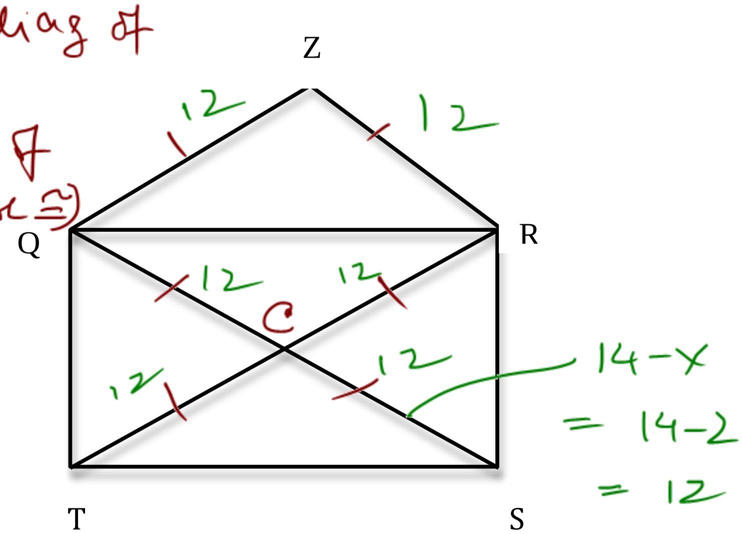
$3 = y$

$RZ = CS$

$6x = 14 - x$

$7x = 14$

$x = 2$



Example 5: Find the measure of LN. Given LI = 3x-2 and MI = 2x +3 and LMNP is a rectangle.

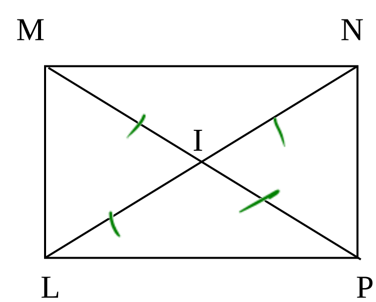
$LI = MI$

$3x - 2 = 2x + 3$

$x = 5$

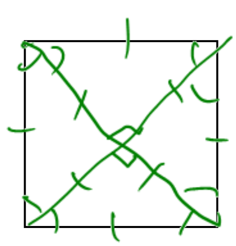
$LI = 3(5) - 2 = 13$

$\therefore LN = 2(LI) = 2(13) = 26$



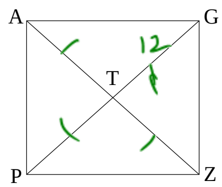
Square

Definition: A square is a rectangle that has two adjacent sides congruent.



Corollary 4.3.3: All sides of a square are congruent.

Example 6:
AGZP is a square with $\overline{GT} = 12$. Find $\overline{AZ}$.



$AT = GT = TZ = TP = 12$

$\therefore AZ = 2(12) = 24$

Rhombus

Definition: A rhombus is a parallelogram with two congruent adjacent sides.

Corollary 4.3.4: All sides of a rhombus are congruent.

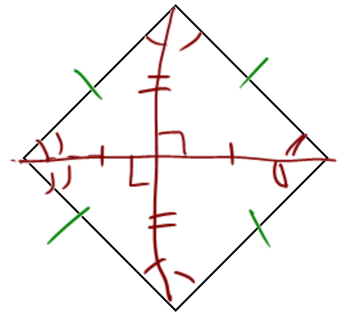
Theorem 4.3.5: The diagonals of a rhombus are perpendicular.

<u>Squares and Rhombi</u>
A square is a quadrilateral with 4 right angles and 4 congruent sides.

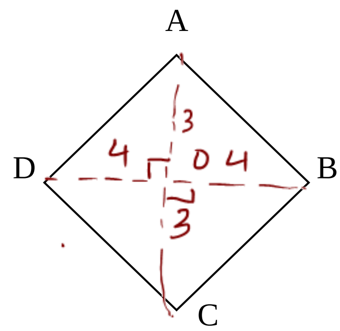
A rhombus is also a quadrilateral, but its characterized by 4 congruent sides; a rhombus does NOT have four congruent angles.

The properties of a parallelogram apply to both squares and rhombi. A rhombus however has two special properties:

2) The diagonals of a rhombus are perpendicular (they form right angles)
3) Each diagonal of a rhombus bisects a pair of opposite angles (the angles are cut in half).



Example 7: Given a rhombus ABCD



a. If DC = 6.3 , find the perimeter of ABCD.

Perimeter = 4(6.3) = 25.2

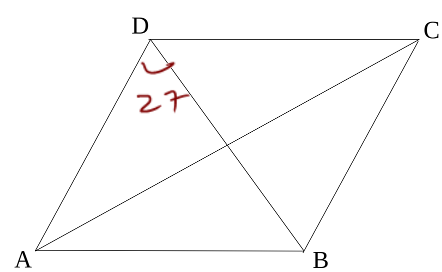
b. If DB = 8 and AC = 6, find DC.

Since diagonal bisect eachother ∴ DO = 4 OC = 3

$DC^2 = DO^2 + OC^2$
$= 4^2 + 3^2 = 16 + 9 = 25$

$DC^2 = 25$
$DC = \sqrt{25} = 5$

Example 8:



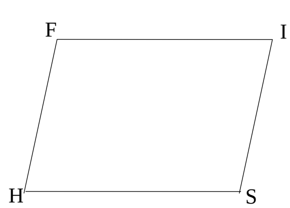
ABCD is a rhombus. m∠ADB = 27. Find the m∠ADC.

$m\angle ADC = 2\, m\angle ADB$
$= 2(27)$
$= 54$

$m\angle DAB + m\angle ADC = 180$
$m\angle DAB = 180 - m\angle ADC = 180 - 54 = 136^\circ$

Example 9:

FISH is a rhombus with $\overline{FI}$ = 6x + 2 and $\overline{SI}$ = 8x - 4. Find $\overline{FH}$.

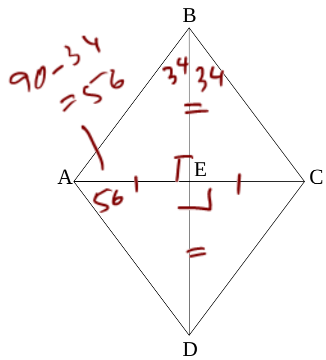


FI = SI
6x + 2 = 8x − 4
6 = 2x
3 = x
FC = 6(3) + 2 = 20

FI = SI = SH = FH ∴ FH = 20

Example 10:

Use rhombus ABCD and the given information to find each value.



a. $\overline{AE} = 14$
find $\overline{AC}$

$AC = 2(AE) = 2(14) = 28$

b. $m\angle ABE = 34°$
find $m\angle ABC$

$m\angle ABC = 2\ m\angle ABE$
$= 2(34) = 68°$

$m\angle BAD = 180 - 68$
$= 112°$

c. find $m\angle DEA$

$= 90$

d. $\overline{CB} = 4x - 1$
$\overline{AB} = 20 + x$
find "x"

$CB = AB$
$4x - 1 = 20 + x$
$3x = 21$
$x = 7$