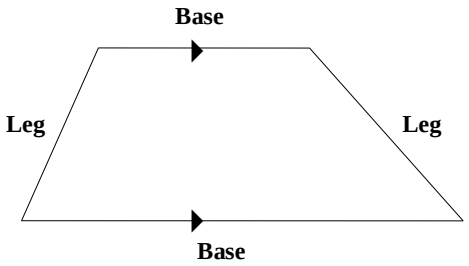
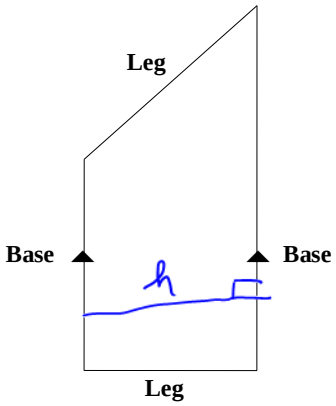
# Trapezoids

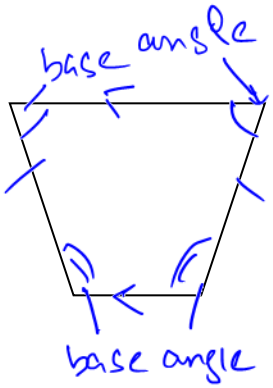
**Definition:** A trapezoid is a quadrilateral with exactly two parallel sides.

Parts of a trapezoid:



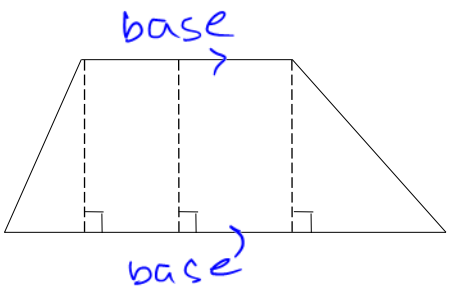

Isosceles Trapezoid:

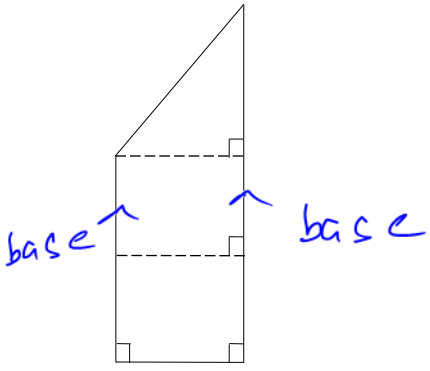



Every trapezoid contains two pairs of consecutive angles that are supplementary.

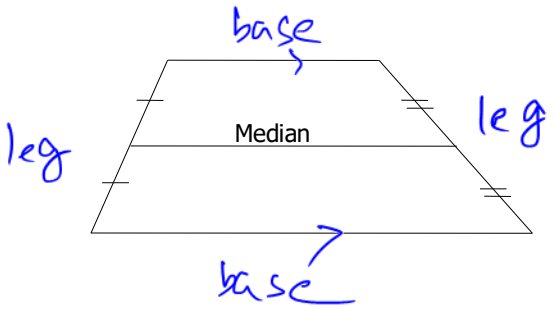
**Definition:**
An **altitude** of a trapezoid is a segment drawn from any point on one of the parallel sides (base) perpendicular to the opposite side (the other base).

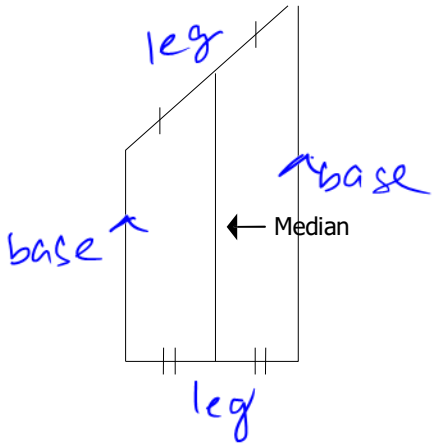
An infinite number of altitudes may be drawn in a trapezoid.



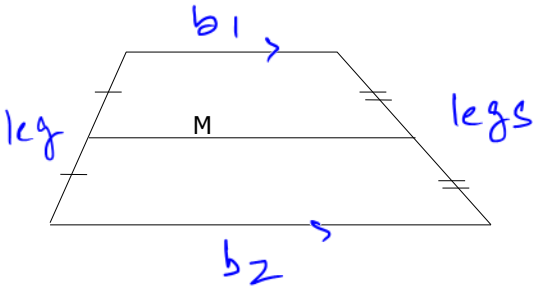



**Definition:**
A **median** of a trapezoid is the segment that joins the midpoints of the nonparallel sides (legs).




**Theorem:** The median of a trapezoid is parallel to each base and the length of the median equals one-half the sum of the lengths of the two bases.




$M = \frac{1}{2}(b_1 + b_2)$ $M \| b_1$ $M \| b_2$

**Definition:**
An **isosceles** trapezoid is a trapezoid in which the legs (nonparallel sides) are congruent.

An isosceles trapezoid features some special properties not found in all trapezoids.

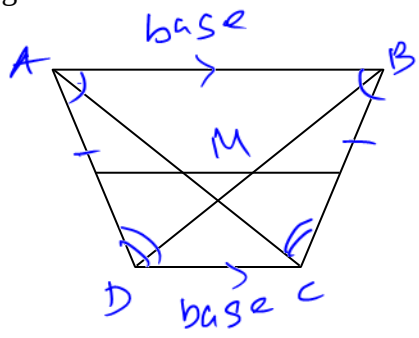
**Theorem 4.4.1:** The base angles of an isosceles trapezoid are congruent.

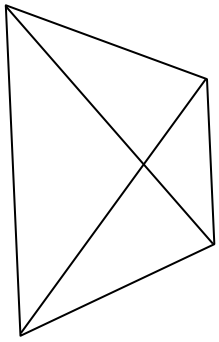
**Theorem 4.4.2:** The diagonals of an isosceles trapezoid are congruent.

**Properties of Isosceles Trapezoid**

1. The legs are congruent. $\overline{AD} \cong \overline{BC}$
2. The bases are parallel. $\overline{AB} \parallel \overline{CD}$
3. The lower base angles of an isosceles trapezoid are congruent. $\angle C \cong \angle D$
4. The upper base angles of an isosceles trapezoid are congruent. $\angle A \cong \angle B$
5. The lower base angle is supplementary to any upper base angle. $m\angle A + m\angle D = 180$, $m\angle B + m\angle C = 180$
6. The diagonals of an isosceles trapezoid are congruent. $\overline{AC} \cong \overline{BD}$
7. The median is parallel to the base. $M \parallel \overline{AB}$, $M \parallel \overline{CD}$
8. The length of the median equals one-half the sum of the lengths of the two bases.

$M = \frac{1}{2}(AB + CD)$



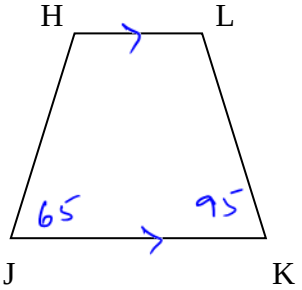
**Proving that Trapezoid is isosceles**

1. If legs of a trapezoid are congruent then it is an isosceles trapezoid.

2. If two base angles of a trapezoid are congruent, then it is an isosceles trapezoid.

3. If the diagonals of a trapezoid are congruent, then it is an isosceles trapezoid.

**Example 1:** Given the trapezoid HLJK




$m\angle H + m\angle J = 180$

$m\angle H = 180 - 65 = 115^\circ$

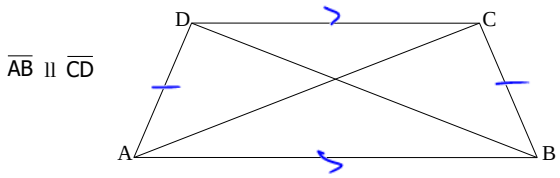
$m\angle L + m\angle K = 180$

$m\angle L = 180 - 95 = 85^\circ$

If the $m\angle J = 65^\circ$ and the $m\angle K = 95^\circ$, the measure of angles H and L .

**Example 2:**

Use Isosceles Trapezoid ABCD with length of AD = BC.




a. $m\angle DAB = 75°$. Find the $m\angle ADC$.

$m\angle DAB + m\angle ADC = 180$

$m\angle ADC = 180 - 75 = 105°$

b. $\overline{AC} = 40$. Find $\overline{BD}$.

$AC = BD$ (as diagonals of isosc trap are $\cong$)

$\therefore BD = 40$

c. If $m\angle A = 6x + 25$ and $m\angle B = 8x + 15$, find the measures of angle C and D.

$m\angle A = m\angle B$ (as lower base $\angle$s of isosc trap are $\cong$)

$6x + 25 = 8x + 15$

$10 = 2x$

$5 = x$

$m\angle A = 6(5) + 25$

$= 30 + 25 = 55°$

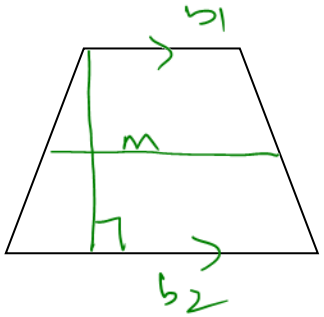
$m\angle B = 55$

$m\angle A + m\angle D = 180$

$m\angle D = 180 - 55 = 125°$

$m\angle D = m\angle C \quad \therefore m\angle C = 125°$

**Definition:** An altitude is a line segment from one vertex of one base of the trapezoid and perpendicular to the opposite base.

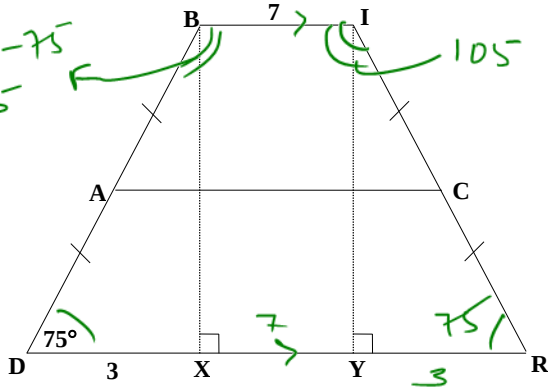



**Theorem 4.4.3:** The length of the median of a trapezoid equals one-half the sum of the bases.

$$m = \frac{1}{2}(b_1 + b_2)$$

**Example 3:**

Find the missing measures of the given trapezoid.




a. m∠IRD = 75

b. YR = 3

c. DR = 3 + 7 + 3 = 13

d. AC = $\frac{1}{2}(BI + DR)$

$= \frac{1}{2}(7+13)$

$= \frac{1}{2}(20) = 10$

BD = IR

∴ BIRD is an isosc trap

180 − 75 = 105

105

In △BDX & △IRY

$\overline{BD} \cong \overline{IR}$

∠BXD ≅ ∠IYR (Both are 90°)

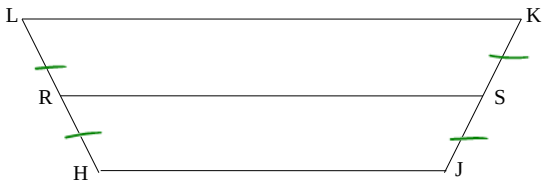
∠BDX ≅ ∠IRY (Base ∠s)

∴ △BDX ≅ △IRY (AAS)

∴ $\overline{DX} \cong \overline{YR}$ (CPCTC)

**Example 4:**

HJKL is an isosceles trapezoid with bases $\overline{HJ}$ and $\overline{LK}$, and median $\overline{RS}$. Use the given information to solve each problem.




a. $\overline{LK}$ = 30
$\overline{HJ}$ = 42
find $\overline{RS}$ $= \frac{1}{2}(LK + HJ)$

$= \frac{1}{2}(30 + 42) = \frac{1}{2}(72) = 36$

b. $\overline{RS}$ = 17
$\overline{HJ}$ = 14
find $\overline{LK}$

$RS = \frac{1}{2}(LK + HJ)$

$17 = \frac{1}{2}(LK + 14)$

$34 = LK + 14 \Rightarrow 20 = LK$

c. $\overline{RS}$ = x + 5
$\overline{HJ}$ + $\overline{LK}$ = 4x + 6
find $\overline{RS}$

$RS = \frac{1}{2}(LK + JH)$

$x + 5 = \frac{1}{2}(4x + 6)$

$2(x+5) = 4x + 6$

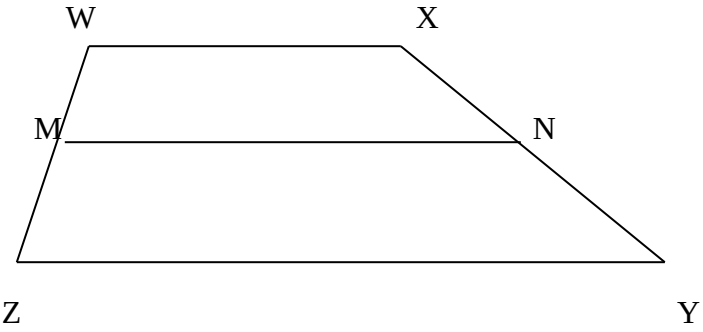
$2x + 10 = 4x + 6$

$4 = 2x \Rightarrow 2 = x$

$RS = 2 + 5$

$= 7$

**Example 5:** Given WXYZ is a trapezoid with $\overline{WX} \parallel \overline{ZY}$, $\overline{MN}$ is the median




a. If WX =19 and ZY = 31, find MN

$MN = \frac{1}{2}(WX + ZY)$

$= \frac{1}{2}(19 + 31) = \frac{1}{2}(50) = 25$

b. If WX = 4x – 7, MN = 2x + 10 and ZY = 2x + 1, find x and the lengths of WX, MN and ZY.

$MN = \frac{1}{2}(WX + ZY)$

$2x + 10 = \frac{1}{2}(4x - 7 + 2x + 1)$

$2(2x + 10) = (4x - 7 + 2x + 1)$

$4x + 20 = 6x - 6$

$26 = 2x$

$13 = x$

$WX = 4(13) - 7 = 45$

$MN = 2(13) + 10 = 36$

$ZY = 2(13) + 1 = 27$

SUMMARY CHARTS:

| Special Quadrilateral | Diagonals Are Always | | Diagonals Always Bisect | |
|---|---|---|---|---|
| | Congruent | Perpendicular | Each Other | Angles |
| **Parallelogram** | **No** | **No** | **Yes** | **No** |
| **Rectangle** | **Yes** | **No** | **Yes** | **No** |
| **Rhombus** | **No** | **Yes** | **Yes** | **Yes** |
| **Square** | **Yes** | **Yes** | **Yes** | **Yes** |
| **Trapezoid** | **No** | **No** | **No** | **No** |
| **Isosceles Trapezoid** | **Yes** | **No** | **No** | **No** |

There is an excellent chart in your book on page 205.