Ratios, Rates and Proportions

RATIOS:

The ratio of two numbers (a and b) may be written in a variety of ways. For example:

$\frac{a}{b}$ a ÷ b   a to b $\underline{a:b}$

In writing the ratio of two numbers, it is usually helpful to express the ratio (fraction) in simplest form. For example, the ratio of 50 to 100 is expressed as follows:

$$\frac{50}{100} = \frac{1}{2}$$

1:2

3 & 6 $\quad \frac{3}{6} = \frac{1}{2} \quad$ 1:2

3:5 $\quad \frac{3}{5}, \frac{6}{10}, \frac{30}{50}, \frac{3000}{5000}$

Example 1:

If the measure of ∠A is 60° and ∠B is a right angle, find the ratio of the m∠A to m∠B.

order in which it is mentioned in very important

$$\frac{m\angle A}{m\angle B} = \frac{60}{90} = \frac{2}{3} \text{ (or 2:3)}$$

ratio of m∠B to m∠A $= \frac{m\angle B}{m\angle A} = \frac{90}{60} = \frac{3}{2}$

$= 3:2$

$$\frac{100 miles}{10 gallons} = \text{10 mpg.}$$

PROPORTIONS:

An equation that states that two ratios are equal is called a proportion. The following are examples of a proportion:

$$\frac{3}{6} = \frac{1}{2} \qquad \frac{24}{16} = \frac{3}{2}$$

In the above examples, if you cross multiply each proportion you will get the same number on both sides of the equal sign. This is always true with a proportion. A proportion can also contain variables.

Example 2:

$\frac{3}{6} = \frac{1}{2}$ $3 \times 2 = 6 \times 1$ $6 = 6$

$\frac{x}{3} = \frac{14}{21}$ $21x = 42$ $x = 2$

$$\frac{x}{3} = \frac{14}{21} \Rightarrow 21 \cdot \frac{x}{3} = \frac{14}{\cancel{21}}\cancel{21} \Rightarrow \frac{21x}{3} = 14 \Rightarrow \frac{21x}{\cancel{3}}\cancel{3} = 14 \cdot 3$$

$$\Rightarrow 21x = 42$$

Property 1 (means-extremes)

In a proportional the product of the means equal the product of the extremes: that is if $\frac{a}{b} = \frac{c}{d}$ (where $b \neq 0$ and $d \neq 0$) then $a \bullet d = c \bullet \cancel{d}\, b$

Example 3:

Solve for x using Property 1: $\frac{9}{45} = \frac{21}{x}$

$9x = 45 \cdot 21$

$x = \frac{\overset{5}{\cancel{45}} \cdot 21}{\cancel{9}} = 105$

Example 4:

Solve for x using Property 1: $\frac{x+5}{9} = \frac{x-1}{3}$

$3(x+5) = 9(x-1)$

$3x + 15 = 9x - 9$

$15 + 9 = 9x - 3x$

$24 = 6x \Rightarrow 4 = x$

Property 2: In a proportion the means and extremes (or both the means and extremes) maybe interchanged so that if $\frac{a}{b} = \frac{c}{d}$,(where a, b, c, and d are non- zero) then $\frac{a}{b} = \frac{c}{d}$, $\frac{d}{b} = \frac{c}{a}$, and $\frac{d}{c} = \frac{b}{a}$

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{d}{b} = \frac{c}{a}$

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{d}{b} = \frac{c}{a} \Rightarrow \frac{d}{c} = \frac{b}{a}$

Property 3:

If $\frac{a}{b} = \frac{c}{d}$ (where $b \neq 0$ and $d \neq 0$), then $\frac{a+b}{b} = \frac{c+d}{d}$, $\frac{a-b}{b} = \frac{c-d}{d}$

$\frac{a}{a+b} = \frac{c}{c+d}$ $\qquad \frac{a+b}{a-b} = \frac{c+d}{c-d}$ ||

$\frac{a-b}{b} = \frac{d-c}{d}$ X

Example 5:

1. $\frac{6}{8} = \frac{3}{4}$ $\qquad \frac{6+8}{8} = \frac{3+4}{4} \Rightarrow \frac{14}{8} = \frac{7}{4}$

2. $\frac{6}{8} = \frac{3}{4}$ $\qquad \frac{6-8}{8} = \frac{3-4}{4} \Rightarrow \frac{-2}{8} = -\frac{1}{4}$

$\frac{6+8}{6-8} = \frac{3+4}{3-4} \Rightarrow \frac{14}{-2} = \frac{7}{-1}$

The geometric mean between two positive numbers, a and b, is the positive number, x, where:

$$\frac{a}{x} = \frac{x}{b} \Rightarrow x^2 = ab$$

Example 6: Find the geometric mean of 4 and 9.

let x be the geometric mean of 4 & 9

$$\frac{4}{x} = \frac{x}{9}$$

$$x^2 = 4 \cdot 9 = 36$$

$$x = \sqrt{36} = \pm 6$$

Example 7: find the measure of the three angles of a triangle if the measures of the angles have a ratio 1: 2: 3

let x be the common factor

1st $\angle = x$

2nd $\angle = 2x$

3rd $\angle = 3x$

$\angle 1 + \angle 2 + \angle 3 = 180$

$x + 2x + 3x = 180$

$6x = 180$

$x = 30$

$m\angle 1 = 30$

$m\angle 2 = 60$

$m\angle 3 = 90$

4 : 5 : 7

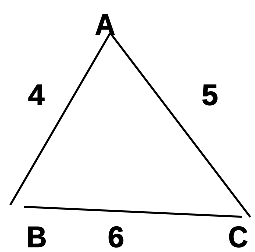
$4x + 5x + 7x = 180$

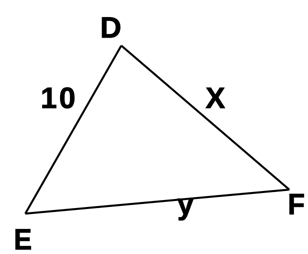
$16x = 180$

$x =$

Example 8: ΔABC and ΔDEF have the following properties:

$\overline{AC} \leftrightarrow \overline{DF}$





$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{4}{10} = \frac{6}{y} = \frac{5}{x}$$

$$\frac{4}{10} = \frac{6}{y}$$

$4y = 6 \cdot 10$

$y = \frac{6 \cdot 10}{4} = 15$

$$\frac{4}{10} = \frac{5}{x}$$

$4x = 5 \cdot 10$

$x = \frac{5 \cdot 10}{4} = 12.5$

Example 9: A recipe calls for 4 eggs and 3 cups of milk. To prepare for a larger number of guests, a cook uses 14 eggs. How many cups of milk are needed?

$\frac{\text{egg}}{\text{milk}}$

let x milk will be needed

$$\frac{4}{3} = \frac{14}{x}$$

$4 \cdot x = 14 \cdot 3$

$x = \frac{14 \cdot 3}{4}$

$= 10.5$ cups