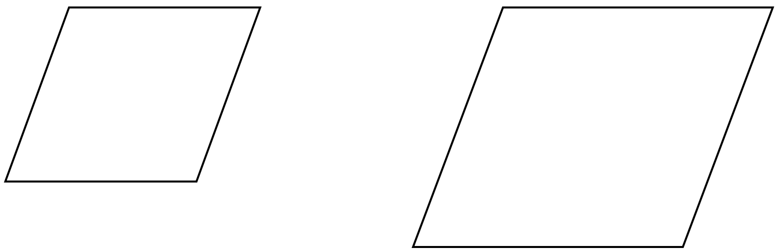
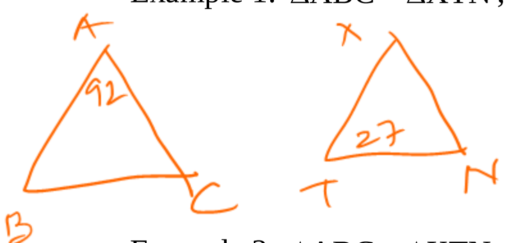
Two geometric figures that have exactly the same shape are similar ~

Definition: Two polygons are similar if and only if two conditions are satisfied:

1. All parts of corresponding angles are congruent.
2. All pairs of corresponding sides are proportional.



Example 1: $\Delta ABC \sim \Delta XTN$, $m\angle A = 92°$, $m\angle T = 27°$, find the measures of the other angles.



$m\angle A = m\angle X = 92^0$

$m\angle B = m\angle T = 27°$

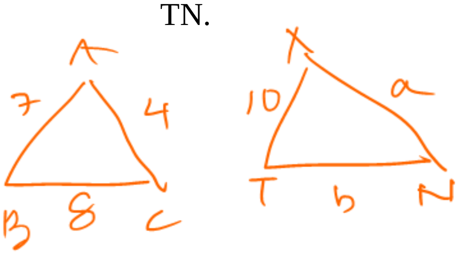
$m\angle C = m\angle N = 61°$

$m\angle A + m\angle B + m\angle C = 180$

$92 + 27 + m\angle C = 180$

$m\angle C = 61$

Example 2: $\Delta ABC \sim \Delta XTN$, if AB = 7, AC = 4, BC = 8 and XT = 10. Find the length of XN and TN.



$\frac{AB}{XT} = \frac{BC}{TN} = \frac{AC}{XN}$

$\frac{7}{10} = \frac{8}{b} = \frac{4}{a}$

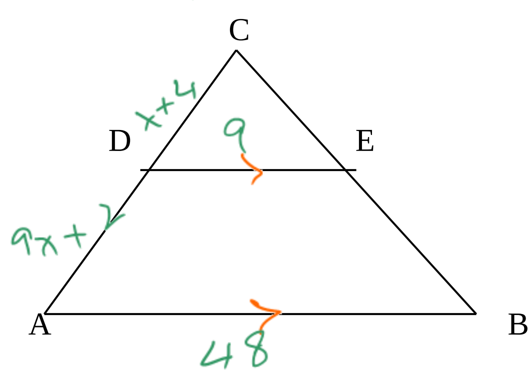
$\frac{7}{10} = \frac{8}{b}$

$7b = 8 \cdot 10 \Rightarrow b = \frac{8 \cdot 10}{7} = \frac{80}{7}$

$\frac{7}{10} = \frac{4}{a} \Rightarrow 7a = 4 \cdot 10$

$a = \frac{40}{7}$

Example 3: $\Delta ABC \sim \Delta DEC$ and AB || DE, solve for x. Given: $DC = x + 4$, $AD = 9x + 2$, $DE = 9$ and $AB = 48$



$\frac{DE}{AB} = \frac{DC}{AC}$

$\frac{9}{48} = \frac{x+4}{(9x+2)+(x+4)}$

$\frac{9}{48} = \frac{x+4}{10x+6}$

$9(10x+6) = 48(x+4)$

$90x + 54 = 48x + 192$

$90x - 48x = 192 - 54$

$41x = 138 \Rightarrow x = \frac{138}{42}$

Example 4: On a blueprint the length of an 18 foot room is represented by a line segment that is 3.6 inches long. What would a 15 foot room be represented by?

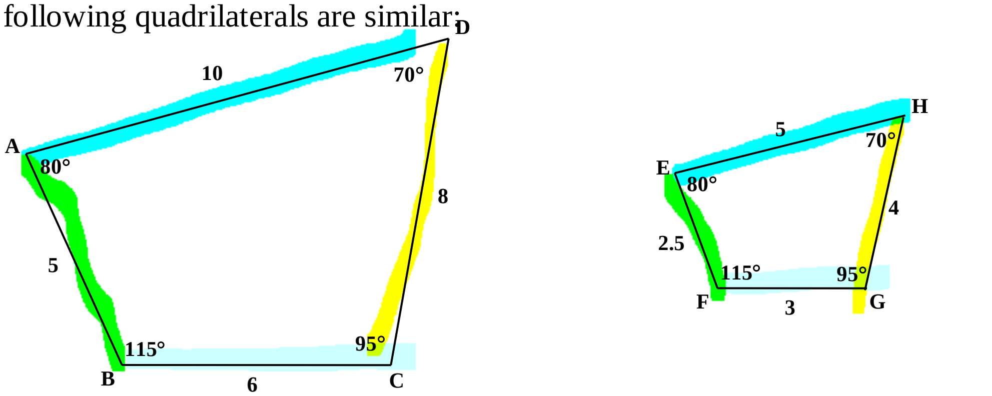
BL	3.6	x
AL	18	15

$$\frac{3.6}{18} = \frac{x}{15}$$

$$\frac{(3.6)(15)}{18} = x \Rightarrow 3 \text{ inches} = x$$

Scale Factor: The ratio of the lengths of two corresponding sides of two similar polygons.

The following quadrilaterals are similar:



Why are they similar? Because……..

1) $\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$

2) $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} = \frac{2}{1}$ ⟵ **This is the scale factor!**

Example 5: Complete each statement: RSTU ~ EFGH

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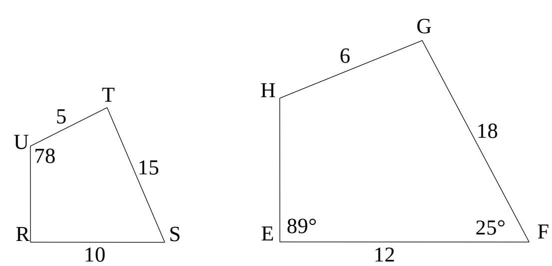


1. $\angle R =$ $\angle E$
2. $\angle S =$ $\angle F$
3. $\angle H =$ $\angle U$
4. $\angle G =$ $\angle T$
5. $\frac{HG}{UT} = \frac{6}{5}$
6. $\frac{ST}{FG} = \frac{15}{18} = \frac{5}{6}$

Example 6: determine the height of the Eiffel Tower if a person is 5.5 feet tall casts a .5 foot shadow and the Eiffel Tower casts a 90 foot shadow at the same time.

	SH	AH
ET	90	x
M	0.5	5.5

$$\frac{90}{0.5} = \frac{x}{5.5}$$

$$x = \frac{(90)(5.5)}{0.5}$$

$$= \frac{(90)(\cancel{5.5}\,55)}{\cancel{0.5}\,5} = 990$$

feet

Two congruent polygons are also similar.

Question:

Two similar polygons are always congruent, true or false?

Example 7:

Which figures must be similar?

a. Any two isosceles triangles

No, even 2 Δ's are isos the LS may not be congruent.

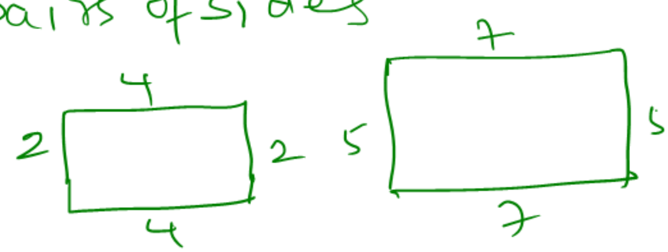
b. Any two regular pentagons

Yes, all angles are 108^0 and sides are proportional

c. Any two rectangles

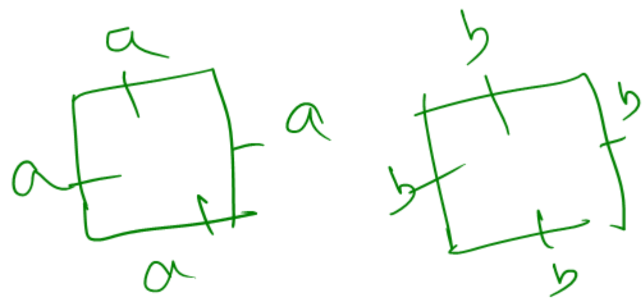
No, angles are 90^0 but pairs of sides are not in prop.

$\frac{2}{5}, \frac{4}{7}$



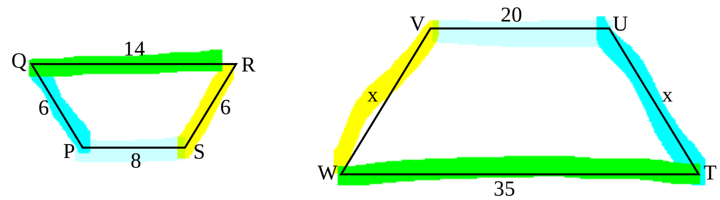
d. Any two squares

Yes all angles are 90^0 and all pairs of sides are prop.



$\frac{a}{b}$

Example 8: Trapezoid PQRS is similar to trapezoid UTWV. Find the value of x.



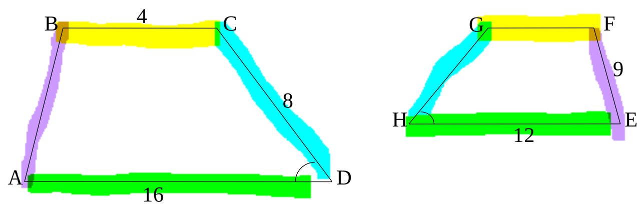
a. identify the scale factor

$$\frac{PS}{UV} = \frac{8}{20} = 2/5$$

b. UT or x =

$$\frac{PS}{UV} = \frac{RS}{VW} \Rightarrow \frac{8}{20} = \frac{6}{x} \Rightarrow x = \frac{(6)(20)}{8} = 15$$

Example 9: ABCD ~ EFGH, they are both quadrilaterals.



$$\text{Scale factor} = \frac{AD}{EH} = \frac{16}{12} = \frac{4}{3}$$

a. Find AB.

$$\frac{4}{3} = \frac{AB}{FE} \Rightarrow \frac{4}{3} = \frac{AB}{9} \Rightarrow \frac{\cancel{4}\cdot \cancel{9}^{3}}{\cancel{3}} = AB \Rightarrow 12 = AB$$

b. Find HG.

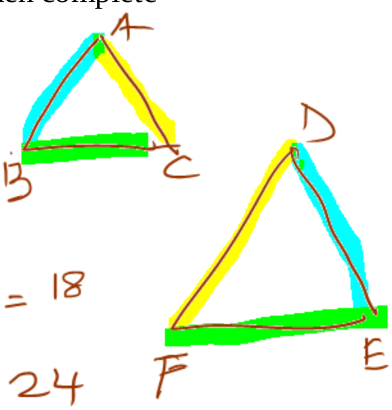
$$\frac{4}{3} = \frac{CD}{HG} \Rightarrow \frac{4}{3} = \frac{8}{HG} \Rightarrow HG = \frac{\cancel{8}^{2}\cdot 3}{\cancel{4}} = 6$$

c. Find FG.

$$\frac{4}{3} = \frac{BC}{FG} \Rightarrow \frac{4}{3} = \frac{4}{FG} \Rightarrow FG = \frac{\cancel{4}\cdot 3}{\cancel{4}} = 3$$

Example 10:
ΔABC~ΔDEF. The scale factor of ΔABC to ΔDEF is 3/7. Draw a picture and then complete each statement.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$



a. If AB=15, then DE=

$$\frac{3}{7} = \frac{AB}{DE} \Rightarrow \frac{3}{7} = \frac{15}{DE} \Rightarrow DE = \frac{15.7}{3} = 35$$

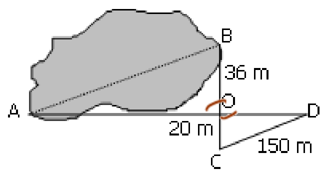
b. If EF=42, then BC=

$$\frac{3}{7} = \frac{BC}{EF} \Rightarrow \frac{3}{7} = \frac{BC}{42} \Rightarrow BC = \frac{3.42}{7} = 18$$

c. If DF=56, then AC=

$$\frac{3}{7} = \frac{AC}{DF} \Rightarrow \frac{3}{7} = \frac{AC}{56} \Rightarrow AC = \frac{3.56}{7} = 24$$

Example 11: In order to find the distance AB across a lake, a surveyor constructed ΔOCD similar to ΔOBA. He measured OB (36m), OC (20m), and CD (150m) directly to obtain the lengths shown. Find the length of AB.



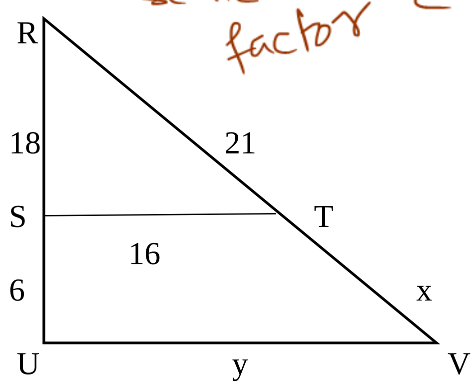
$$\frac{OC}{OB} = \frac{CD}{AB}$$

$$\boxed{\frac{OC}{OB} = \frac{CD}{AB} = \frac{OD}{OA}}$$

$$\frac{20}{36} = \frac{150}{AB}$$

$$AB = \frac{15\cancel{0} \cdot \overset{18}{\cancel{36}}}{\cancel{2}\cancel{0}} = 270\,m$$

Example 12: ΔRST ~ ΔRUV find x and y



scale factor ←

$$\frac{RS}{RU} = \frac{ST}{UV} = \frac{RT}{RV}$$

$$\frac{18}{24} = \frac{16}{y} = \frac{21}{21+x}$$

$$\frac{18}{24} = \frac{16}{y}$$

$$\frac{3}{4} = \frac{16}{y}$$

$$3y = 16 \cdot 4$$

$$y = \frac{16 \cdot 4}{3} = \frac{64}{3}$$

$$\frac{18}{24} = \frac{21}{21+x}$$

$$\frac{3}{4} = \frac{21}{21+x}$$

$$3(21+x) = 21 \cdot 4$$

$$21+x = \frac{\overset{7}{\cancel{21}} \cdot 4}{\cancel{3}}$$

$$21+x = 28$$

$$x = 7$$