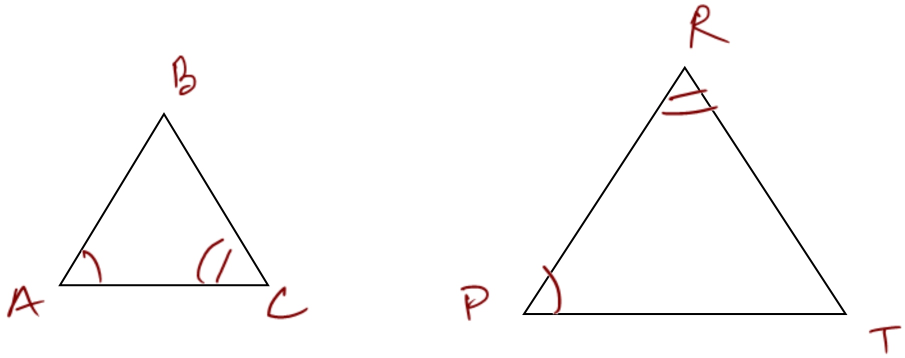
Postulate 15: If the three angles of one triangle are congruent to the three angles of a second triangle, then the triangles are similar (AAA).

Corollary 5.3.1: If two angles of one triangle are congruent to the two angles of another triangle, then the triangle are similar (AA).

(AA):
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example 1:
Given: $\angle A \cong \angle P$,
$\angle C \cong \angle R$



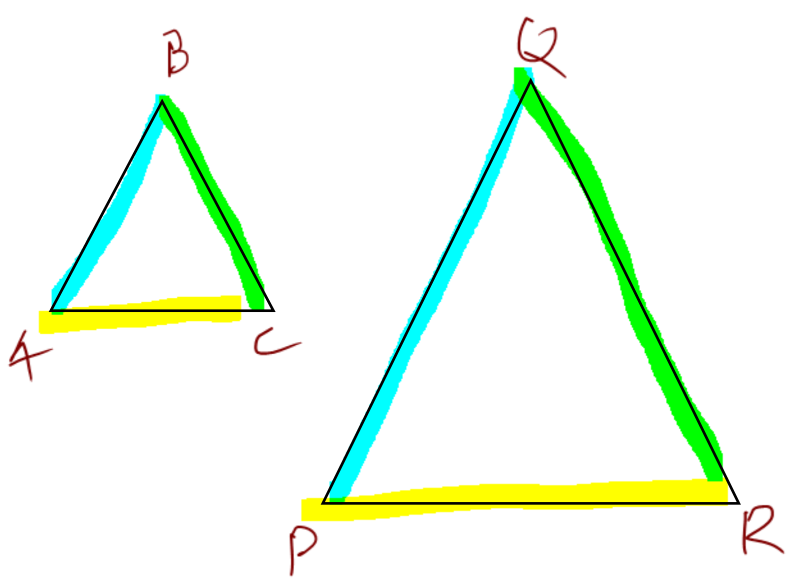
Conclusion:

$\triangle ABC \sim \triangle PTR$

(SSS):
If each side of one triangle and the corresponding side of another triangle are proportional, then the triangles are similar.

Example 2:

Given $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$



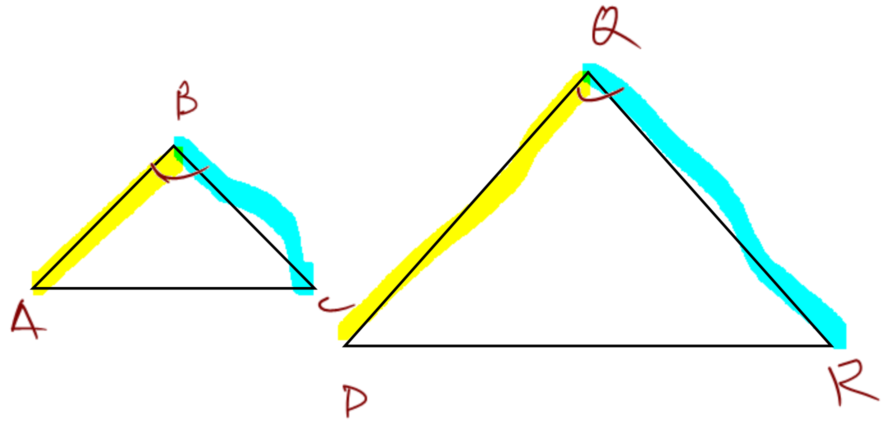
Conclusion:

$\triangle ABC \sim \triangle PQR$

(SAS):
If the measures of two sides of one triangle are proportional to the corresponding sides of another triangle AND the included angles are congruent, then the triangles are similar.

Example 3:

Given: $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ and $\angle B \cong \angle Q$



Conclusion:

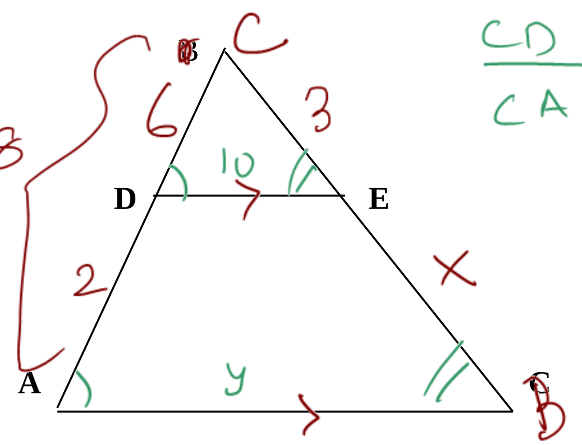
$\triangle ABC \sim \triangle PQR$

Example 4: In the figure below, $AB \parallel DE$, $DA = 2$, $CA = 8$, and $CE = 3$. Find CB.

In $\triangle CDE$ & $\triangle CAB$

$\angle D \cong \angle A$
$\angle E \cong \angle B$ } corres $\angle$s

$\therefore \triangle CDE \sim \triangle CAB$ (AA)



$\dfrac{CD}{CA} = \dfrac{DE}{AB} = \dfrac{CE}{CB}$

$\dfrac{CD}{CA} = \dfrac{CE}{CB}$

$\dfrac{6}{8} = \dfrac{3}{3+x}$

$6(3+x) = 3 \cdot 8$
$18 + 6x = 24$
$6x = 6$
$x = 1$

$CB = 3 + 1 = 4$

RULES:

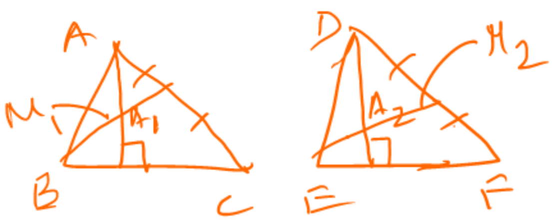
1) **If two triangles are similar, then the** perimeters are proportional **to the measures of corresponding sides.**

2) **If two triangles are similar, then the measures of the corresponding** altitudes (form 90°) are proportional **to the measures of the corresponding sides.**

3) **If two triangles are similar, then the measures of the** corresponding angle bisectors **of the triangles** are proportional **to the measures of the corresponding sides.**

4) **If two triangles are similar, then the measures of the corresponding** medians are proportional **to the measures of the corresponding sides.**

$\triangle ABC \sim \triangle DEF$



ABC — P_1, A_1, M_1

DEF — P_2, A_2, M_2

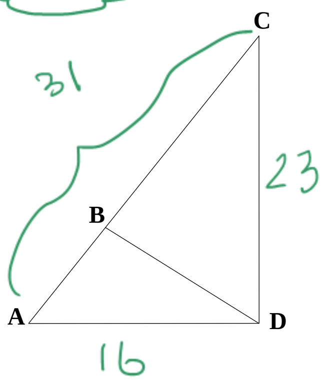
$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF} = \dfrac{P_1}{P_2} = \dfrac{A_1}{A_2} = \dfrac{M_1}{M_2}$

$P_1 \to$ Perimeter of ΔABD

$P_2 \to$ " " ΔADC

Example 5:

$\Delta ABD \sim \Delta ADC$. If AD=16, AC=31, and DC=23, find the perimeter of ΔABD.



$\Delta ABD \to$ / $\Delta ADC \to$

$$\frac{AB}{AD} = \frac{BD}{DC} = \frac{AD}{AC} = \frac{P_1}{P_2}$$

$$\frac{AD}{AC} = \frac{P_1}{P_2}$$

$$P_2 = 31 + 23 + 16 = 70$$

$$\frac{16}{31} = \frac{P_1}{70}$$

$$70\left(\frac{16}{31}\right) = P_1$$

CSSTP: Corresponding sides of similar triangles are proportional.

CASTC: Corresponding angles of similar triangles are congruent.

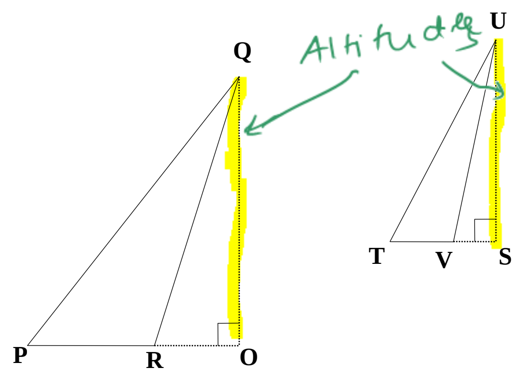
Theorem 5.3.2: The lengths of the corresponding altitudes of similar triangles have the same ratio as the lengths of any pair of corresponding sides.

Example 6:

$\Delta PQR \sim \Delta TUV$. IF QO is an altitude of ΔPQR, and US is an altitude of ΔTUV, then complete the following:

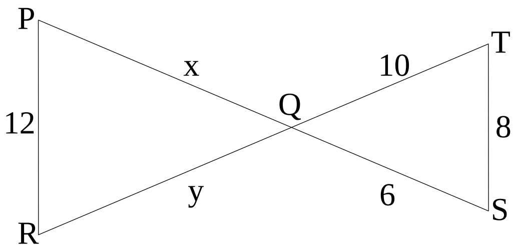
$$\frac{QO}{US} = \frac{QR}{?}\ UV$$

$$\frac{QO}{US} = \frac{?}{TU}\ PQ$$



Example 7: Find the value of x and y.

$\Delta PQR \sim \Delta SQT$



$\frac{PQ}{SQ} = \frac{QR}{QT} = \frac{PR}{ST}$

$\frac{x}{6} = \frac{y}{10} = \frac{12}{8}$

$\frac{x}{6} = \frac{12}{8}$

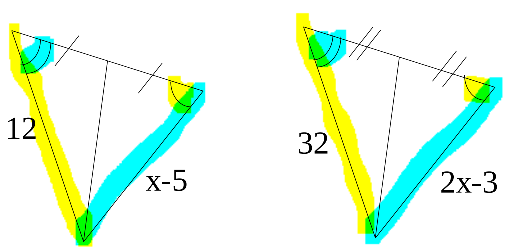
$x = \frac{\cancel{6}^{3} \cdot \cancel{12}^{3}}{\cancel{8}\,\cancel{4}} = 9$

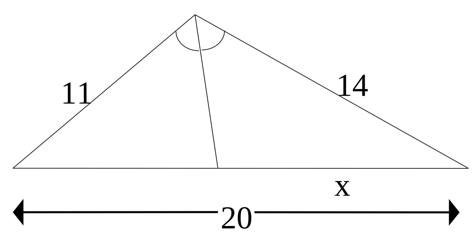
$\frac{y}{10} = \frac{12}{8}$

$y = \frac{\cancel{10}^{5} \cdot \cancel{12}^{3}}{\cancel{8}_{2}} = 15$

Example 8:
a.
x = ________

b) x = ________





$\frac{12}{32} = \frac{x-5}{2x-3}$

$12(2x-3) = 32(x-5)$

$24x - 36 = 32x - 160$

$160 - 36 = 32x - 24x$

$124 = 8x \Rightarrow 15.5 = x$

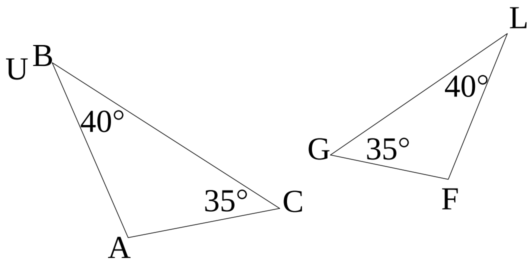
Theorem 5.3.3: (SAS ~) If an angle of one triangle is congruent to an angle of a second triangle and the pairs of sides including the angles are proportional, then the triangles are similar.

Theorem 5.3.4:(SSS ~) If the three sides of one triangle are proportional to the three consecutive sides of one second triangle, then the triangles are similar.

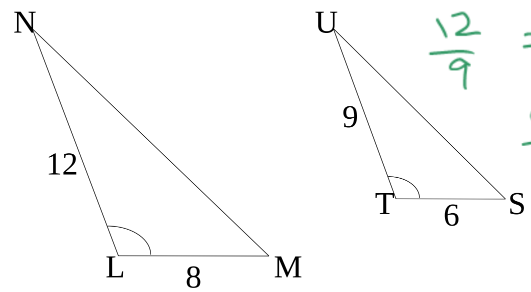
Example 9:

Determine if the triangles are similar and give a reason for your answer (AA, SSS, SAS).

a. similar? Yes
reason AA



b. reason Yes similar? SAS

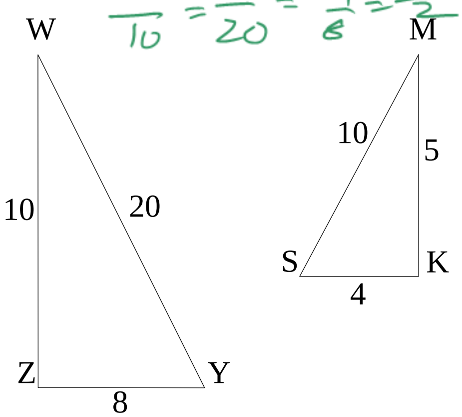


$\frac{12}{9} = \frac{4}{3}$

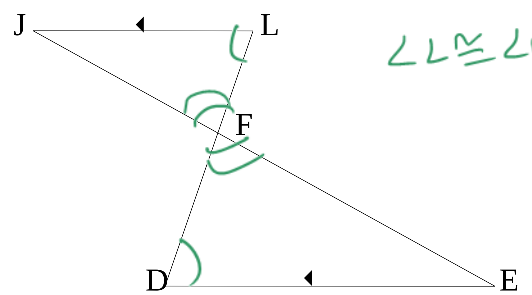
$\frac{8}{6} = \frac{4}{3}$

c. similar? Yes
reason SSS

$\frac{5}{10} = \frac{10}{20} = \frac{4}{8} = \frac{1}{2}$



d. similar? Yes reasson A-A

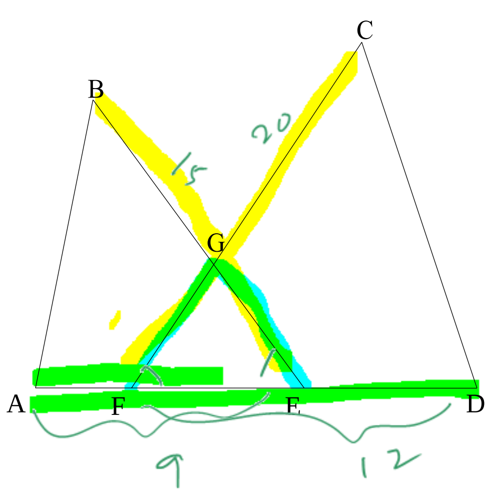


$\angle L \cong \angle D$ Int Alt $\angle s$

Example 10: In the figure below, $FG \cong EG$, $BE = 15$, $CF = 20$, $AE = 9$, $DF = 12$. Determine which triangles in the figure are similar.

$\boxed{\triangle AEB \sim \triangle DFC \ (SAS)}$

$m\angle CFE = m\angle BEF$



$\frac{AE}{DF} = \frac{9}{12} = \frac{3}{4}$

$\frac{BE}{CF} = \frac{15}{20} = \frac{3}{4}$

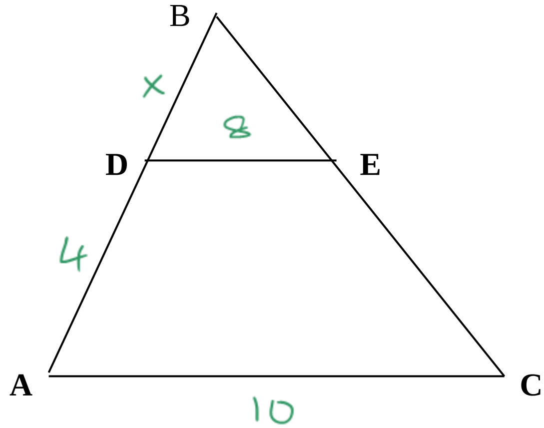
$\therefore \frac{AE}{DF} = \frac{BE}{DF}$

$\therefore AEB \sim \triangle DFC \ (SAS)$

Lemma 5.3.5: If a line segment divides two side of a triangle proportionally, then this line segment is parallel to the third side.

Example 11:
Given ΔABC ~ ΔDBE

If AC = 10 , DE = 8, AD = 4.
Find DB



$$\frac{x}{4} = \frac{8}{10}$$

$$\frac{AB}{DB} = \frac{BC}{BE} = \frac{AC}{DE}$$

$$\frac{AB}{BD} = \frac{AC}{DE}$$

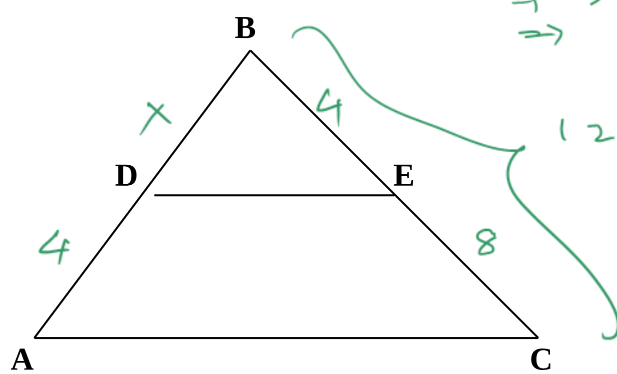
$$\frac{4+x}{x} = \frac{10}{8}$$

$$8(4+x) = 10x \Rightarrow 32 + 8x = 10x$$
$$\Rightarrow 32 = 2x$$
$$\Rightarrow 16 = x$$

Example 12:
Given ΔABC ~ ΔDBE

CB = 12, CE = 8, AD =4.

Find BD



$$\frac{AB}{DB} = \frac{BC}{BE} = \frac{AC}{DE}$$

$$\frac{AB}{DB} = \frac{BC}{BE}$$

$$\frac{x}{4} = \frac{4}{8}$$

$$\frac{4+x}{x} = \frac{12}{4}$$

$$4(4+x) = 12x$$
$$16 + 4x = 12x$$
$$16 = 8x$$
$$2 = x$$