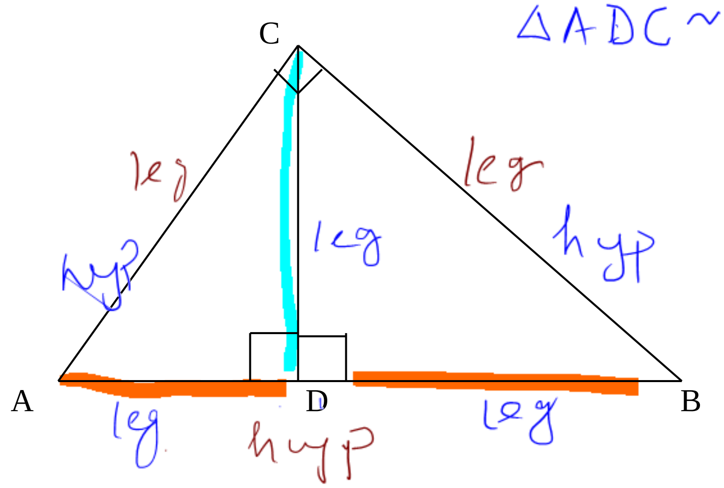
Theorem 5.4.1: The altitude drawn to the hypotenuse of a right triangle separates the right triangle into two right triangles that are similar to each other and to the original right triangle.

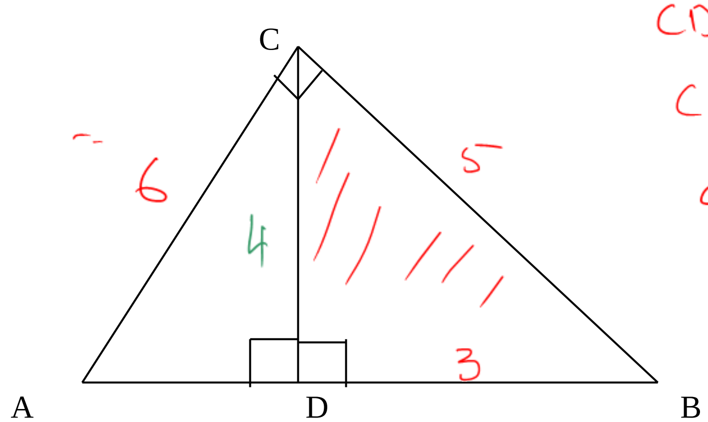
$\triangle ADC \sim \triangle BDC \sim \triangle ACD$



Theorem 5.4.2: The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

$$\frac{AD}{CD} = \frac{CD}{DB} \Rightarrow CD^2 = AD \cdot DB$$

Example 1:
Given a right triangle ABC with altitude DC:



$$CD^2 + DB^2 = CB^2$$
$$CD^2 + 3^2 = 5^2$$
$$CD^2 + 9 = 25$$
$$CD^2 = 16$$
$$CD = 4$$

If BD =3, BC = 5, AC = 6, find DC and AD

$$\frac{AD}{CD} = \frac{CD}{DB} \Rightarrow \frac{AD}{4} = \frac{4}{3}$$
$$\Rightarrow AD = \frac{16}{3}$$

Lemma 5.4.3: The length of each leg of a right triangle is the geometric mean of the length of the segment of the hypotenuse adjacent to that leg. Use figure 5.21page 235:

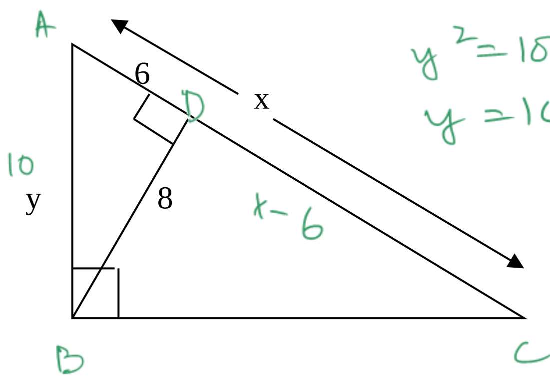
$$\frac{\mathbf{AB}}{\mathbf{AC}} = \frac{\mathbf{AC}}{\mathbf{AD}}$$

Theorem 5.4.4: (Pythagorean Theorem) The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs.

$c^2 = a^2 + b^2$ **c is the longest side or the hypotenuse and this theorem only works with right triangles.**

Example 2:

Find "x" and "y".



$AB^2 = BD^2 + AD^2$

$y^2 = 8^2 + 6^2$

$= 64+36$

$y^2 = 100$

$y = 10$

$BD^2 = AD \cdot DC$

$8^2 = 6 \cdot (x-6)$

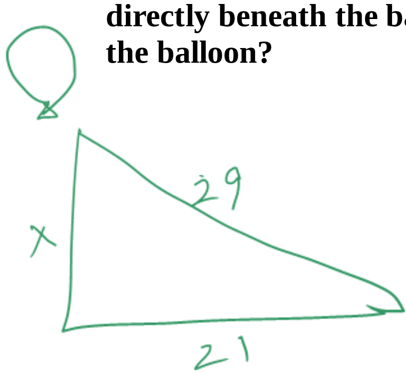
$64 = 6x - 36$

$100 = 6x$

$\frac{100}{6} = x$

Example 3:

A hot air balloon is held in place by the ground crew at a position that is 21 feet from the point directly beneath the balloon. If the rope is of length 29 feet, how far above the ground level is the balloon?



$x^2 + 21^2 = 29^2$

$x^2 = 29^2 - 21^2$

$= (29+21)(29-21)$

$= (50)(8)$

$= 400$

$x^2 = 400$

$x = 20$

$x^2 + 441 = 841$

$x^2 = 400$

$x = 20$

Definition: The Pythagorean triple is a set of three numbers (a, b, c) for which.

$$a^2 + b^2 = c^2$$

Theorem 5.4.7: Let a, b and c represent the lengths of the three sides of the triangle with length c the length of the longest side.

1. **If $c^2 > a^2 + b^2$, then the triangle is obtuse and the angle lies opposite the side of length c.**
2. **If $c^2 < a^2 + b^2$, then the triangle is acute.**

Example 4: Determine the type of triangle represented if the lengths if it sides are as follows:

a^2		b^2	?	c^2

a. **a= 1.5, b = 2 and c =2.5**

Right

$(1.5)^2 + (2)^2 \quad (2.5)^2$

$2.25 + 4 \quad 6.25$

$6.25 = 6.25$

b. **a = 5, b = 7 and c = 9**

Obtuse

$(5)^2 + 7^2 \quad (9)^2$

$25 + 49 \quad 81$

$74 < 81$

c. **a = 10, b = 12 and c = 16**

Obtuse

$10^2 + 12^2 \quad 16^2$

$100 + 144 \quad 256$

$244 < 256$

d. **a = 6, b = 7 and c = 8**

Acute

$6^2 + 7^2 \quad 8^2$

$36 + 49 \quad 64$

$85 > 64$

Example 5:

What is the length of a side of a square with a diagonal length of 10?
Draw a diagram.

$x^2 + 10x - 144 = 0$ $\qquad$ $ax^2 + bx + c = 0$

$a = 1 \quad b = 10 \quad c = -144$ $\qquad$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(-144)}}{2 \cdot 1}$$

$$= \frac{-10 \pm \sqrt{100 + 576}}{2}$$

$$= \frac{-10 \pm \sqrt{676}}{2} = \frac{-10 \pm 26}{2} = \frac{-10+26}{2}, \frac{-10-26}{2}$$

$$= \frac{16}{2}$$

$$= 8$$

$1.x^2 + 10x - 144 = 0$ $\qquad$ $1 \cdot 144 = 144$

$12 \cdot 12$

$3 \cdot 48$

$18 \cdot 8$

$\underbrace{x^2 + 18x}\underbrace{- 8x - 144} = 0$

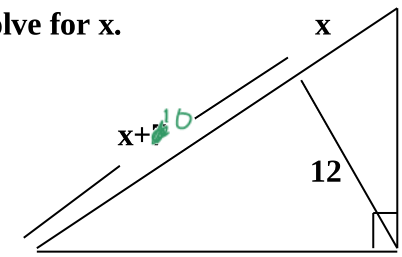
$x(x+18) - 8(x+18) = 0$

$(x+18)(x-8) = 0$

$x + 18 = 0 \qquad x - 8 = 0$

$x = -18 \qquad x = 8$

Example 6: Solve for x.



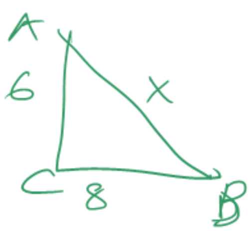
$12^2 = x(x+10)$

$144 = x^2+10x$

$0 = x^2+10x-144$

$x^2+10x-144=0$

Example 7: Given a right triangle with right angle C, AC = 6 and CB = 8. Find the length of AB.



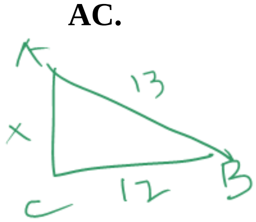
$x^2 = 6^2+8^2$

$= 36+64$

$x^2 = 100$

$x = 10$

Example 8: Given a right triangle with right angle C, AB = 13 and CB = 12. Find the length of AC.



$x^2+12^2=13^2$

$x^2+144=169$

$x^2=25$

$x=5$

Example 9: Determine the type of triangle represented if the length of its sides are as follows:

a b c $\qquad a^2 + b^2 \; ? \; c^2$

a. 3, 5, 7 $\qquad 3^2 + 5^2 \quad 7^2$

$9 + 25 \quad 49$

obtuse $\qquad 34 < 49$

b. 5, 12, 13 $\qquad 25 + 144 \quad 169$

Right $\qquad 169 = 169$

c. 7, 8, 9 $\qquad 7^2 + 8^2 \quad 9^2$

Acute $\qquad 49 + 64 \quad 81$

$113 > 81$

d. 2, 6, 9

$2+6=8<9$

$\therefore$ Not possible to be a $\triangle$