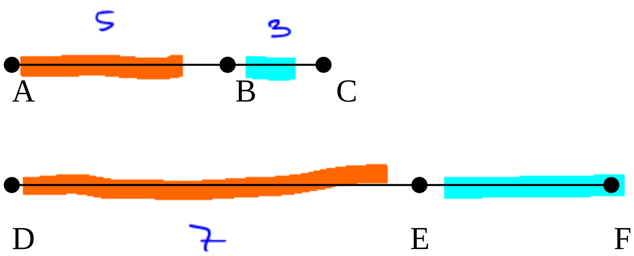
Segments Divided Proportionally

Divided proportionally:

Given line segments:



$\overline{AC}$ and $\overline{DE}$ are divided proportionally at the points B and E respectively.

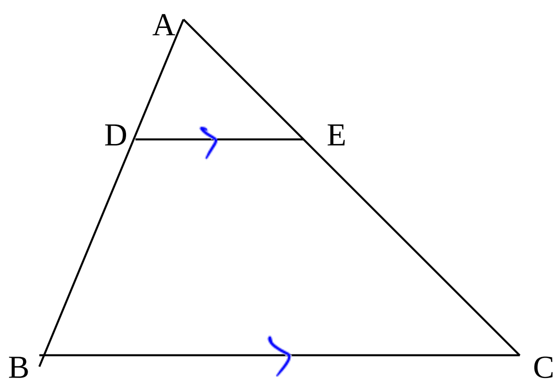
$\dfrac{AB}{DE} = \dfrac{BC}{EF}$ or $\dfrac{AB}{BC} = \dfrac{DE}{EF}$

Example 1: Use the figure form above. If AB = 5, BC = 3 and DE =7, find EF.

$$\frac{AB}{BC} = \frac{DE}{EF} \Rightarrow \frac{5}{3} = \frac{7}{EF}$$

$$\Rightarrow EF = \frac{7 \cdot 3}{5} = \frac{21}{5} = 4.2$$

Theorem 5.6.1: If a line is parallel to one side of a triangle and intersects the other two sides, then it divides these sides proportionally.



$$\frac{\mathbf{AD}}{\mathbf{AE}} = \frac{\mathbf{DB}}{\mathbf{EC}}$$

$$\frac{AD}{BD} = \frac{AE}{EC}$$

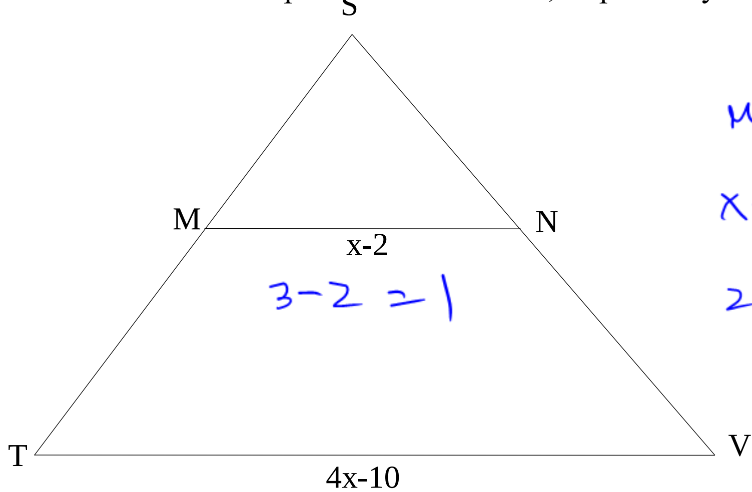
Example 2: Use the figure from above. D and E divide $\overline{AB}$ and $\overline{AC}$ proportionally. If AD = 6, DB = 8 and EC = 10. Find AE.

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$$\frac{6}{8} = \frac{AE}{10} \Rightarrow \frac{6 \cdot 10}{8} = AE \Rightarrow 7.5 = AE$$

Example 3: Problem from 4.2. Notice the difference.

. Points “M” and “N” are midpoints of ST and SV, respectively. Find “x”, MN, and TV.



$3-2=1$

$4(3)-10=2$

$MN = \frac{1}{2} TV$

$x-2 = \frac{1}{2}(4x-10)$

$2(x-2) = 4x-10$

$2x-4 = 4x-10$

$10-4 = 4x-2x$

$6 = 2x$

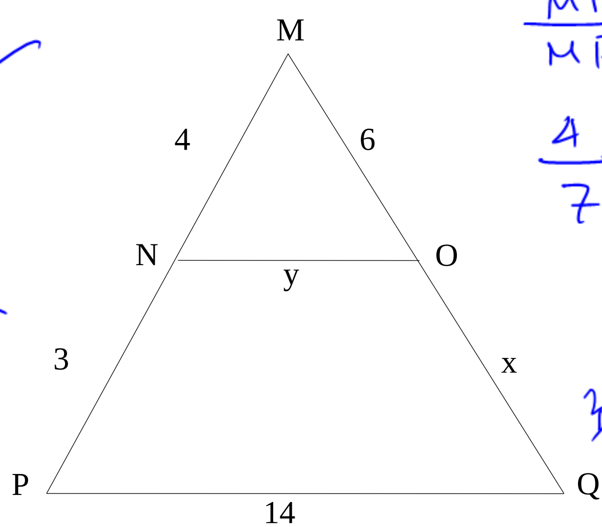
$3 = x$

Example 4:

ΔMNO ~ ΔMPQ. Find the values of “x” and “y”.

$\frac{4}{3} = \frac{6}{x}$ ✓

$\frac{4}{3} = \frac{y}{14}$ ✗



$\frac{MN}{MP} = \frac{NO}{PQ} = \frac{MO}{MQ}$

$\frac{4}{7} = \frac{y}{14} = \frac{6}{x+6}$

$\frac{4}{7} = \frac{y}{14}$

$\frac{\cancel{14}^{2} \cdot 4}{\cancel{7}} = y$

$8 = y$

$\frac{4}{7} = \frac{6}{x+6}$

$4(x+6) = 6 \cdot 7$

$4x+24 = 42$

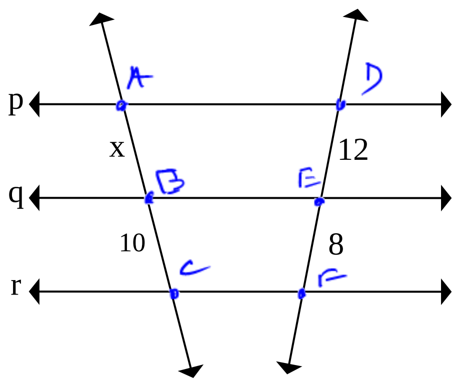
$4x = 18$

$x = \frac{18}{4} = 4.5$

Corollary 5.6.2: when three (or more) parallel lines are cut by a pair of transversals, the transversals are divided proportionally by the parallel lines.

Example 5:

A. Find "x". p ll q ll r.

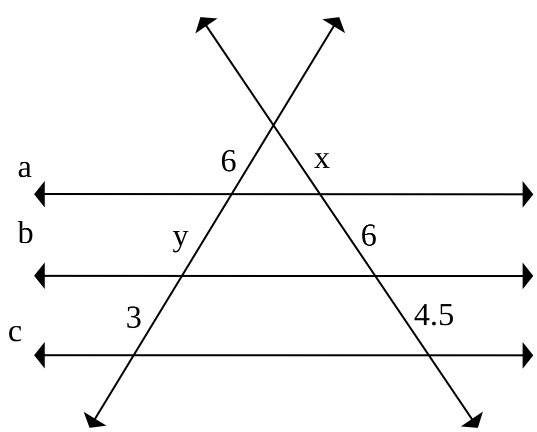


$$\frac{AB}{BC} = \frac{DE}{EF}$$

$$\frac{x}{10} = \frac{12}{8}$$

$$x = \frac{12 \cdot 10}{8} = 15$$

B. Find "x" and "y". a ll b ll c .



$$\frac{AB}{BC} = \frac{AE}{EF} \qquad \frac{BC}{CD} = \frac{EF}{FG}$$

$$\frac{y}{3} = \frac{6}{4.5}$$

$$y = \frac{6 \cdot 3}{4.5} = 4$$

$$\frac{6}{3} = \frac{x}{4.5}$$

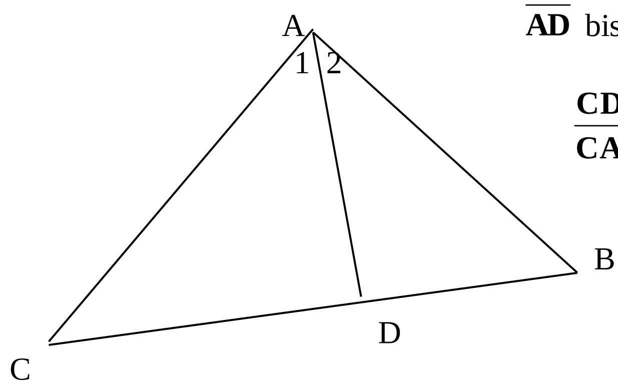
$$\frac{6}{y} = \frac{x}{6}$$

Any

$$\frac{(6)(4.5)}{3} = x$$

$$9 = x$$

Theorem 5.6.3: (The Angle Bisect Theorem) If a ray bisects one angle of a triangle, then it divides the opposite side into proportional segments whose lengths are proportional to the lengths of the to sides that form the bisected angle.

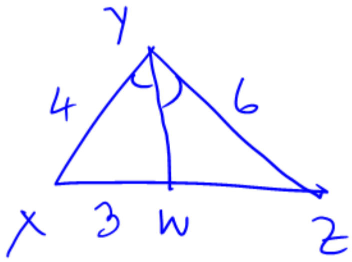


$\overline{\mathbf{AD}}$ bisects $\angle\mathbf{CAB}$ so $\mathbf{m\angle 1 = m\angle 2}$ then

$$\frac{\mathbf{CD}}{\mathbf{CA}} = \frac{\mathbf{BD}}{\mathbf{AB}}$$

Example 6:

Δ**XYZ** , $\overline{\textbf{YW}}$ bisects $\angle XYZ$ $\cancel{\angle XYW}$, if XY = 4, YZ = 6 and XW = 3. Find WZ.



$$\frac{XW}{XY} = \frac{WZ}{YZ}$$

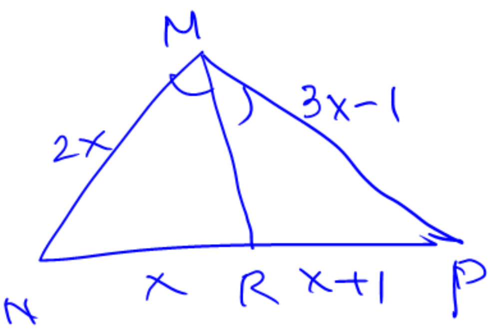
$$\frac{3}{4} = \frac{WZ}{6}$$

$$\frac{3 \cdot 6}{4} = WZ$$

$$4.5 = WZ$$

Example 7:

Δ**PMN** with $\overline{\textbf{MR}}$ bisects $\angle$**NMP**. If MN = 2x, NR = x, RP = $x+1$ and MP = $3x-1$, find x and the measure of $\overline{\text{MP}}$, $\overline{\text{RP}}$, $\overline{\text{MN}}$, and $\overline{\text{NR}}$.



$$\frac{NR}{NM} = \frac{RP}{MP}$$

$$\frac{\cancel{x}}{2\cancel{x}} = \frac{x+1}{3x-1}$$

$$\frac{1}{2} = \frac{x+1}{3x-1}$$

$$3x-1 = 2(x+1)$$

$$3x-1 = 2x+2$$

$$3x-2x = 3$$

$$x = 3$$