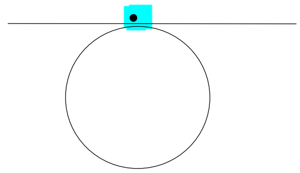
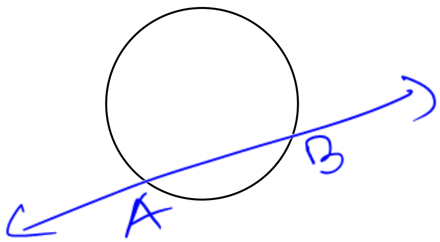
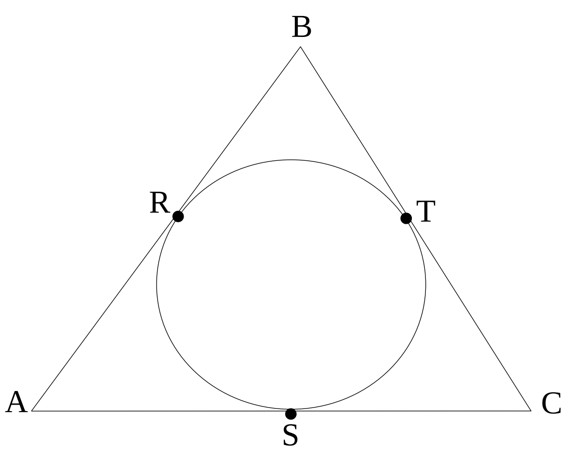
**Definition:** a tangent is a line that intersects a circle at exactly one point, the point of intersection is the point of contact or the point of tangency.

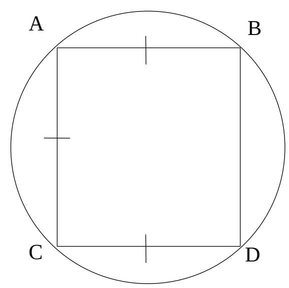


**Definition:** A secant line (or segment or ray) that intersects a circle at exactly two points.

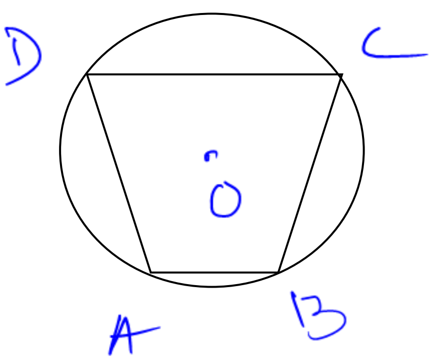



**Definition:** A polygon is inscribed in a circle if its vertices are points on the circle and its sides are chords of the circle. Equivalently, the circle is said to be circumscribed about the polygon. The polygon inscribed in a circle is further described as a cyclic polygon.




**Theorem: 6.2.1:** If a quadrilateral is inscribed in a circle the opposite angles are supplementary. Alternate form: The opposite angles of cyclic quadrilateral are supplementary.

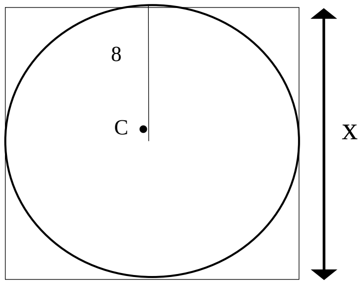



$m\angle B + m\angle D = 180$

$m\angle C + m\angle A = 180$

**Definition:** A polygon is circumscribed about a circle if all sides of the polygon are line segments tangent to the circle also, the circle is said to be inscribed polygon.

**Example 1: Find the value of x**



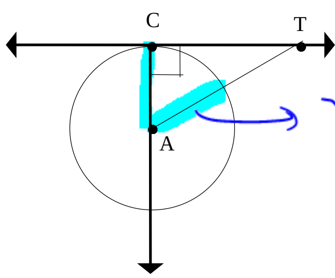

radius = 8

x = diameter

$\therefore x = 16$

**Theorem 6.2.3:** The radius (or any line through the center of the circle) drawn to a tangent at the point of tangency is perpendicular to the tangency point.

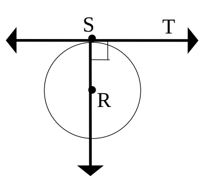
1. If a line is tangent to a circle, then it is also perpendicular to the radius.




radius

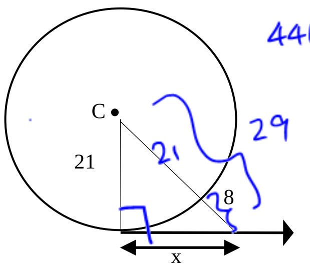
**NOTE:** Because ∠TCA is a right angle then ΔTCA a right triangle. Therefore, you could use the Pythagorean theorem to find the measure of a missing side.

2. If a line is perpendicular to a radius then the line is a tangent of the circle.



**Example 2: Solve for x:**

**a.**



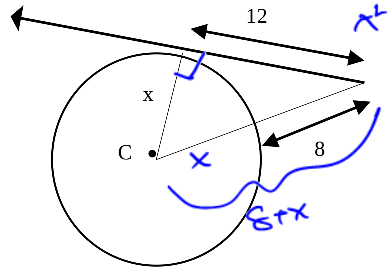

$21^2 + x^2 = 29^2$

$441 + x^2 = 841$

$x^2 = 400$

$x = 20$

**b.**




$x^2 + 12^2 = (8+x)^2$

$x^2 + 144 = 64 + x^2 + 16x$

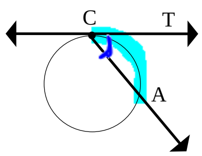
$144 = 64 + 16x$

$80 = 16x$

$5 = x$

RULES:

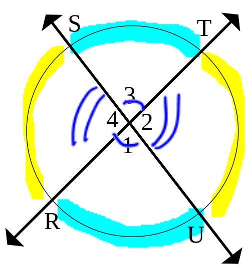
1. If a secant and a tangent intersect at the point of tangency (the place where the tangent "hits" the circle), then the measure of each angle formed is ½ the measure of its intercepted arc.




m∠TCA = ½ m arc CA

**Corollary 6.2.4**

2. If two secants intersect in the interior of a circle, then the measures of an angle formed is ½ the sum of the measures of the arcs intercepted by the angle and its vertical angle.




m∠1 = m∠3 AND m∠2 = m∠4 (ver. opp)

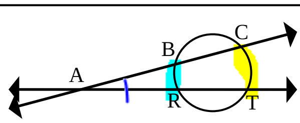
**m∠1** (and m∠3) = ½ **(mRU + mST)**

and

**m∠2** (and m∠4) = ½ **(mTU + mRS)**

**Theorem 6.2.2**

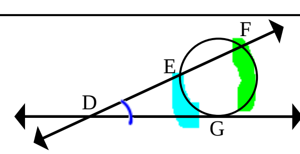
3. If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is ½ the positive difference of the measures of the intercepted arcs. There are three possible cases:




**Two Secants:**

m∠CAT = ½ (mCT - mBR)

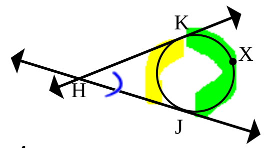
**Theorem 6.2.5**




**A Secant and a Tangent:**

m∠FDG = ½ (FG - EG)

**Theorem 6.2.6**




**Two Tangents:**

m∠KHJ = ½ ($\overset{\frown}{KXJ}$ - $\overset{\frown}{KJ}$)

**Theorem 6.2.7**

**In Summary:**

If the lines intersect **ON** the circle use: angle = $\frac{1}{2}$(arc)

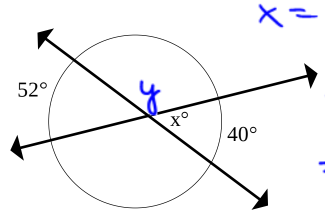
If the lines intersect **IN** the circle use: angle = $\frac{1}{2}$(arc + arc)

If the lines intersect **OUT** of the circle use: angle = $\frac{1}{2}$(big arc – little arc)

**Example 3:**

Find the value of "x".

a.. x = 46



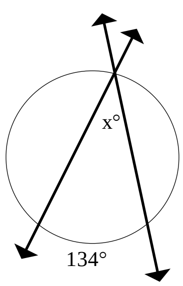

$x = \frac{1}{2}(52 + 40)$

$= \frac{1}{2}(92)$

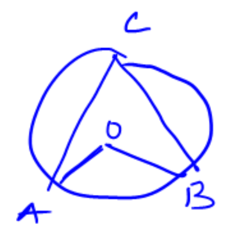
$= 46°$

$y = 180 - x = 180 - 46 = 134°$

b.. x = 67°




$x = \frac{1}{2}(134)$

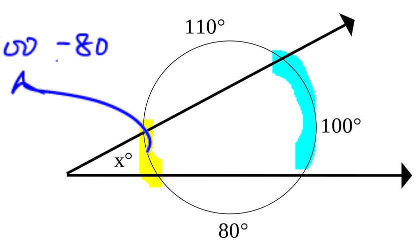



$\angle ACB = \frac{1}{2}\angle AOB$

c.. x = 15°

$360 - 110 - 100 - 80$

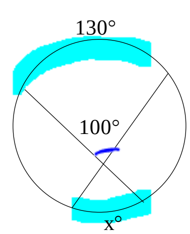
$= 70°$




$x = \frac{1}{2}(100 - 70)$

$= \frac{1}{2}(30) = 15$

d.. x = 70°

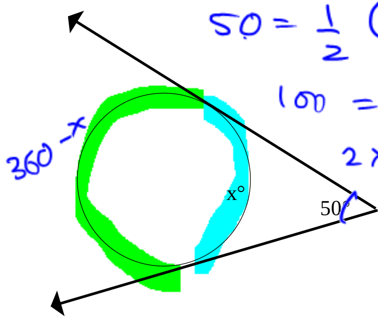



$100 = \frac{1}{2}(130 + x)$

$200 = 130 + x$

$70 = x$

e. $x = $ <u>130</u>



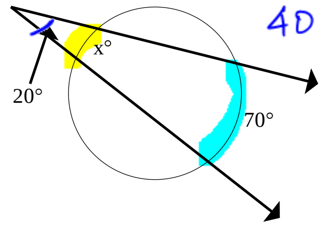

$50 = \frac{1}{2}(360 - x - x)$

$100 = 360 - 2x$

$2x = 260$

$x = 130$

f.. $x = $ <u>30</u>

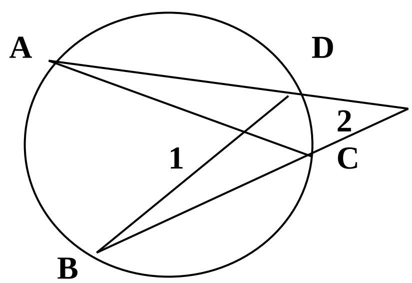



$20 = \frac{1}{2}(70 - x)$

$40 = 70 - x$

$x = 30$

**Example 4: Find the indicated arcs and angles.**




a.

b. **If m of arc AB =** $106°$ **and m arc DC=** $32°$**. Find the measure of** $m\angle 1$ **and** $m\angle 2$**.**

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{DC})$

$= \frac{1}{2}(106 + 32)$

$= \frac{1}{2}(138) = 69°$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{DC})$

$= \frac{1}{2}(106 - 32)$

$= \frac{1}{2}(74) = 37°$

c. **If the m arc AB =** $80°$ **and** $m\angle 1 = 75°$**, find m arc CD.**

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$

$75 = \frac{1}{2}(80 + m\overset{\frown}{CD})$

$150 = 80 + m\overset{\frown}{CD}$

$70 = m\overset{\frown}{CD}$

c. **If the m arc AB =** $88°$ **and** $m\angle 2 = 24°$**, find** $m\angle 1$**.**

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{CD})$

$24 = \frac{1}{2}(88 - m\overset{\frown}{CD})$

$48 = 88 - m\overset{\frown}{CD}$

$m\overset{\frown}{CD} = 40$

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$

$= \frac{1}{2}(88 + 40)$

$= \frac{1}{2}\ 128$

$= 64°$